TOPOGRAPHY-TO-PROJECTION OPTIMIZATION OF STATE-PLANE COORDINATE SYSTEMS: A CASE STUDY FROM THE REPUBLIC OF KOSOVO

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Abstract
This study proposes a methodology for optimizing scale factors in state-plane coordinate systems through integrated analysis of geometric relationships between topography and map projection. Unlike classical approaches that consider only ellipsoid-to-projection distortions, the presented framework evaluates cumulative ground-to-grid differences across topography, geoid, ellipsoid, and map projection surfaces. The optimization is performed through iterative adjustment of the projection scale factor based on the arithmetic mean of signed distortions across the national territory. The method is applied to the Republic of Kosovo, where the existing CRS (Kosovaref01) inherits projection parameters from a legacy system without integrated ground-based optimization. Using a high-resolution analytical grid, an optimized map projection variant with scale factor 1.00012 was developed. The results show that 91.7% of the territory falls within distortions below 100 ppm and 98.3% below 150 ppm. The optimization process produces a territorially balanced distortion structure, in which cumulative positive and negative distortion effects are almost completely compensated across the territory. The proposed methodology substantially outperformed the corresponding classical ellipsoid-based optimization approach, which resulted in a systematically negative distortion structure across the territory. The framework developed in this study provides a generalizable methodology for modern CRS optimization and geodetic applications.

Keywords: *State-plane coordinate system, Ground-to-grid distortion, Linear distortion, Scale factor optimization, Multi-surface analysis*

INTRODUCTION

Coordinate reference systems (CRS) constitute the fundamental infrastructure (ISO, 2019) for geospatial data integration, engineering surveying, cadastral systems, and large-scale mapping. The geometric quality of a CRS is strongly influenced by the design of its map projection component (Maling, 1973), which directly affects the relationship between real-world measurements and their representation in planar coordinate systems (Idrizi, 2014).

Conventional approaches to map projection design are primarily based on the analysis of distortions between the reference ellipsoid and the projection plane (Snyder, 1993). Within this classical framework, projection parameters are selected to minimize scale distortions on the ellipsoid (IES, 2001), which is treated as the principal geometric reference surface. Although this approach has dominated geodetic and cartographic practice for decades, it does not fully reflect the physical reality of spatial measurements (Idrizi et al., 2018).

In practical geodetic applications, objects of interest such as roads, buildings, engineering structures, and cadastral parcels are located on the topographic surface of the Earth rather than on the mathematical reference ellipsoid. Consequently, geometric discrepancies between ground measurements and projected coordinates are influenced not only by projection geometry, but also by cumulative reductions associated with topography, geoid, and ellipsoid transformations (Rollins and Lyle, 2019).

Previous investigations of state-plane coordinate systems in the Balkan region and Kosovo have shown that projection distortions are traditionally evaluated within an ellipsoid-to-projection framework, where the reference ellipsoid is treated as the principal geometric surface (Idrizi et al., 2009; Idrizi, 2020). Subsequent studies emphasized that practical geodetic measurements are performed on the topographic surface and therefore involve cumulative geometric effects associated

with topography, geoid, ellipsoid, and projection transformations (Idrizi, 2014; Idrizi et al., 2018). More recent research has further demonstrated the importance of integrating these surfaces into CRS evaluation and optimization procedures, particularly in high-accuracy geospatial applications (Idrizi, 2025a; Idrizi, 2025c). Similar trends can also be observed in contemporary geodetic practice, where optimization strategies increasingly focus on reducing discrepancies between ground measurements and projected coordinates (Dennis, 2023).

Unlike conventional optimization approaches focused primarily on balancing extreme distortion values derived from ellipsoid-to-projection relationships (Nikolli et al., 2016), the methodology proposed in this study introduces a territorial balancing concept based on the spatial equilibrium of positive and negative ground-to-grid distortions between topography and projection.

In this context, a clear research gap remains: the lack of an integrated and spatially explicit methodology for CRS optimization based on the complete transformation chain from topography to map projection (Idrizi, 2025a). This study addresses this gap by proposing a multi-surface optimization framework in which distortions are evaluated as cumulative ground-to-grid differences and analyzed through their spatial distribution across the territory.

It should be emphasized that the optimization framework is intended for state-plane coordinate systems supporting high-accuracy geospatial applications, where ground-to-grid consistency is directly influenced by topographic and geoid-related effects. In mapping applications where these effects remain smaller than the positional accuracy permitted by the map scale and intended use, the conventional ellipsoid-to-projection optimization framework remains fully adequate. Consequently, the proposed methodology should be regarded as a complementary optimization approach rather than a replacement for classical map projection design.

The methodology is applied to the Republic of Kosovo, where the current CRS (Kosovaref01) inherited its projection parameters from a historical regional framework (Idrizi, 2020) without integrated ground-to-projection optimization (Bajrami Lubishtani et al., 2025). Using a high-resolution analytical grid and iterative scale-factor adjustment, an optimized projection variant is developed and statistically evaluated.

Although the optimization in this study is performed within the current Gauss–Krüger projection framework of Kosovo, the primary objective is not to define the final optimal projection for the territory of the Republic of Kosova, but rather to establish a generalizable optimization methodology applicable to different projection systems and territorial contexts.

The principal methodological novelty of this study lies in reformulating CRS optimization as a territorial balancing problem based on signed ground-to-grid distortions.

THEORETICAL FRAMEWORK AND MULTI-SURFACE MODEL

The geometric relationship between real-world measurements and their representation in planar coordinate systems is established through a sequence of transformations involving multiple reference surfaces. In conventional cartographic and geodetic practice, map projection design is primarily based on distortions (Bandrova and Gyurov, 2024) evaluated between the reference ellipsoid and the projection plane. Consequently, projection parameters are commonly optimized with respect to ellipsoidal geometry rather than real ground conditions (Idrizi, 2025b).

However, geospatial objects such as roads, buildings, engineering structures, and cadastral parcels physically exist on the topographic surface of the Earth. Therefore, the transformation from measured ground distances to projected coordinates involves cumulative geometric reductions across several reference surfaces, including topography, geoid, ellipsoid, and projection (Idrizi, 2025d). This transformation sequence can be expressed as Topography → Geoid → Ellipsoid → Projection (figure 1).

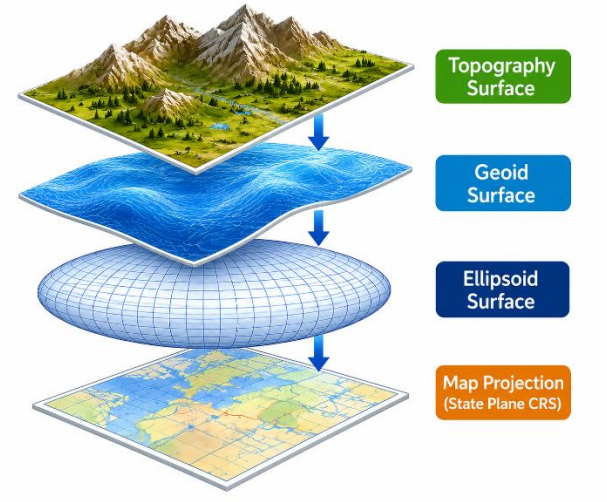


Figure 1. Topography to State-plane Coordinate System approach

Each transformation step introduces geometric discrepancies that contribute cumulatively to the final ground-to-grid distortion. As a result, the total distortion affecting projected coordinates cannot be interpreted solely as an ellipsoid-to-projection effect (Idrizi, 2014), but rather as the combined consequence of the complete transformation chain (Idrizi, 2025c).

The concept of evaluating distortions between topographic and projected space was initially introduced in Idrizi (2014), where the limitations of conventional ellipsoid-based analyses were identified. This approach was subsequently extended in Idrizi et al. (2018), where a regional multi-surface framework for CRS assessment in the Balkan Peninsula was proposed. The present study further develops this methodology through integrated spatial modelling and iterative optimization of projection parameters.

Within this framework, distortions are evaluated as cumulative geometric differences between topographic distances and their projected equivalents. Let L_t denote the horizontal distance measured on the topographic surface and L_p the corresponding distance represented in the map projection plane. The signed linear distortion is expressed as:

$$\Delta L_i = L_{p,i} - L_{t,i}$$

where:

- ΔL_i - represents the local distortion at spatial unit i,
- $L_{t,i}$ - represents the horizontal distance on the topographic surface,
- $L_{p,i}$ - represents the projected distance in map projection plane.

The corresponding relative distortion (δ_i) is expressed as:

$$\delta_i = \frac{L_{p,i} - L_{t,i}}{L_{t,i}} = \frac{\Delta L_i}{L_{t,i}}$$

For practical interpretation, the same relative distortion values are expressed either in parts per million (ppm) or equivalently in centimeters per kilometer (cm/km):

$$\delta_i(ppm) = \delta_i \times 10^6$$

$$\delta_i(cm/km) = \delta_i \times 10^5$$

Unlike conventional approaches that minimize distortions only with respect to the reference ellipsoid, the presented framework treats CRS optimization as a territorial balancing problem based on signed ground-to-grid distortions. In this

formulation, positive and negative distortions compensate each other spatially, producing a geometrically balanced coordinate system with minimal systematic territorial bias.

This approach transforms CRS optimization from a purely projection-based adjustment problem into an integrated territorial optimization model based on real topography-to-map projection relationships.

STUDY AREA AND REFERENCE SYSTEM

The Republic of Kosovo was selected as a case study because its current state-plane coordinate system, Kosovaref01, represents a transition from the historical Hermannskogel/Bessel 1841 framework toward the modern ETRS89/GRS80 reference system (IES, 2001; Mulic, 2018; Idrizi, 2020). Although the geodetic datum was modernized, the projection component remained based on the inherited Gauss–Krüger configuration with central meridian 21°E and scale factor 0.9999 (Idrizi, 2020). Previous analyses demonstrated that these projection parameters were not evaluated using integrated topography-to-projection optimization criteria (Idrizi et al., 2009; Bajrami Lubishtani et al., 2025), making Kosovaref01 an appropriate test environment for evaluating the presented methodology.

The study was conducted over the territory of the Republic of Kosovo. The analysis is based on the national state-plane coordinate system Kosovaref01, defined in the European Terrestrial Reference System 1989 (ETRS89) and realized on the GRS80 reference ellipsoid. The system is officially registered under EPSG code 9141 (Idrizi, 2020).

The projection component of Kosovaref01 is based on the Gauss–Krüger projection with a central meridian at 21°E, a scale factor of 0.9999, and a false easting of 7,500,000 m. These parameters originate from the historical Yugoslav projection framework and currently represent the official state projection system used for surveying, cadastral, engineering, and cartographic applications in Kosovo (Bajrami Lubishtani et al., 2025).

Within the present study, Kosovaref01 serves as the reference system for comparative analysis and evaluation of distortion behavior. The optimization process was performed by systematically modifying the projection scale factor while preserving the remaining projection parameters. This approach enables direct assessment of the influence of scale-factor adjustment on the spatial distribution of ground-to-grid distortions across the national territory.

SPATIAL SAMPLING AND DATA STRUCTURE

To enable spatially explicit evaluation of ground-to-grid distortions, a regular analytical grid with a spatial resolution of 1 km × 1 km was generated over the territory of the Republic of Kosovo (figure 2). The adopted grid structure covers the entire study area and consists of 10,892 spatial units (grid points), representing a homogeneous national sampling framework.

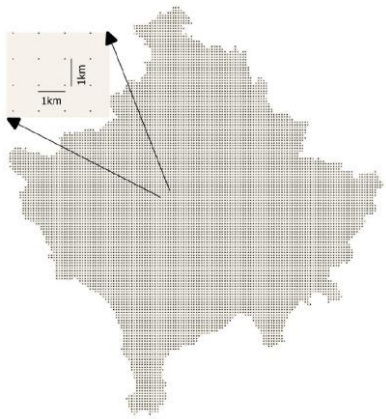


Figure 2. Test model with 10892 points in 1km spatial resolution

The use of a regular grid enables consistent spatial comparison of distortion values and allows the identification of regional distortion patterns across the territory. Unlike analyses based only on selected control points or administrative units, the adopted approach provides continuous and statistically balanced spatial coverage.

For each grid point, the following geodetic and geometric parameters were determined:

- geographic coordinates,

- projection-cartesian coordinates,
- ellipsoidal height,
- geoid height,
- orthometric height and terrain elevation,
- ground-to-grid distortion values.

Topographic information was derived from high-resolution elevation datasets, while geoid-related (Idrizi, 2013) reductions were incorporated through gravity-related modelling components (Bajrami Lubishtani et al., 2020).

The selected spatial resolution represents a balance between computational efficiency and spatial representativeness. A $1km$ sampling interval enables sufficient sensitivity for detecting regional distortion variations while maintaining manageable computational complexity for nationwide analysis.

Because all spatial units represent equal territorial components of the analytical grid, each unit contributes equally to the optimization process and statistical evaluation of distortion behavior across the territory.

PROJECTION OPTIMIZATION

The optimization of the coordinate system was performed by iteratively modifying the projection scale factor within the current CRS of Kosova based on a Gauss–Krüger projection framework. For each tested scale factor k_j, ground-to-grid distortions were recomputed for all spatial units of the analytical grid. The objective was to identify the projection scale factor that minimizes the absolute value of the mean signed ground-to-grid distortion over the territory, i.e., for which the arithmetic mean approaches zero. For each spatial unit i, the signed linear distortion is defined as:

$$\Delta L_i(k_j) = L_{p,i}(k_j) - L_{t,i}$$

where:

- $L_{p,i}(k_j)$ - represents the projected distance computed using the tested projection scale factor k_j, and
- $L_{t,i}$ - represents the horizontal distance on the topographic surface.

The corresponding relative distortion is expressed as:

$$\delta_i(k_j) = \frac{\Delta L_i(k_j)}{L_{t,i}}$$

or, in ppm and cm/km:

$$\delta_i(k_j)_{ppm} = \delta_i(k_j) \times 10^6$$

$$\delta_i(k_j)_{cm/km} = \delta_i(k_j) \times 10^5$$

The mean value of signed relative distortions over the analytical grid is expressed as:

$$\bar{\delta}(k_j) = \frac{1}{n}\sum_{i=1}^{n} \delta_i\,(k_j) \to 0$$

where:

- $\bar{\delta}(k_j)$ - represents the mean signed distortion corresponding to the tested scale factor k_j, and

- n - represents the total number of spatial units in test model.

This condition represents the fundamental territorial balancing criterion of the proposed optimization model, where positive and negative distortion effects tend toward spatial equilibrium across the territory. Because all spatial units of the analytical grid represent equal territorial areas, minimization of the mean value of signed distortions simultaneously implies minimization of the cumulative territorial imbalance between areas affected by positive and negative distortions.

The optimization criterion is based on signed distortions rather than absolute values. Therefore, the optimal scale factor is determined by minimizing the mean value of signed distortions across all spatial units:

$$k_{\text{opt}} = \arg\min_{k_j} \left| \frac{1}{n} \sum_{i=1}^{n} \delta_i \left(k_j\right) \right|$$

where:

- n - represents the total number of spatial units in test model,
- $\delta_i(k_j)$ - represents the signed relative ground-to-grid distortion corresponding to the tested scale factor.

Because each spatial unit represents an equal territorial component of the analytical grid, this condition expresses the territorial equilibrium between positive and negative distortion effects. The absolute operator is applied only to the mean value of signed distortions in order to evaluate its deviation from zero, while individual distortion values retain their original sign throughout the optimization process.

Consequently, the optimization process does not seek to minimize absolute distortions independently, but rather to identify the projection scale factor that produces the most spatially balanced distortion structure across the territory.

Whereas conventional projection optimization methods determine the projection scale factor through balancing extreme distortions derived from ellipsoid-to-projection relationships, the methodology developed in this study performs optimization through territorial equilibrium of signed distortions between topography and the projection plane.

The optimization variable is the projection scale factor, while the optimization criterion is the minimization of the absolute value of the mean signed ground-to-grid distortion over the territory. Territorial equilibrium is not imposed as an independent condition but emerges naturally when the arithmetic mean of signed distortions approaches zero.

RESULTS

The analysis of the current state-plane coordinate system, Kosovaref01, reveals a non-uniform distribution of distortions across the national territory (Idrizi, 2025a, Idrizi et al., 2018, Bajrami Lubishtani et al., 2025). The results indicate that no part of the territory falls within the low-distortion range below 100 ppm. Most of the territory is concentrated within distortion ranges between 150 and 250 ppm, while high-distortion zones exceeding 250 ppm occupy approximately 22.8% of the territory. Maximum distortions reach approximately 500 ppm in localized areas, indicating a systematic spatial bias in the existing projection configuration (table 1, table 2, figure 3 and figure 4a).

The iterative optimization converged to an exact mathematical scale factor of k = 1.0001213671716. At this value, the cumulative territorial imbalance between positive and negative distortion effects is reduced to approximately 8 cm² over the entire territory of the Republic of Kosovo, while the corresponding mean signed distortion over the 1 km analytical grid is approximately 0.0019 mm/km (0.0019 ppm). These values are negligible for nationwide CRS optimization. Therefore, for practical CRS implementation, the optimized scale factor was generalized to k = 1.00012.

The optimized projection variant with k=1.00012 produces a substantially different distortion structure. The tested scale factor was iteratively adjusted until the mean value of signed distortions approached zero. The results show that approximately 91.7% of the territory falls within distortion ranges below 100 ppm, while approximately 98.3% remains below 150 ppm. At the same time, areas exceeding 250 ppm are reduced to approximately 0.1% of the territory, and the maximum observed distortions are significantly lower than those of the existing system (table 1, table 2, figure 3 and figure 4b).

These results demonstrate a clear redistribution of distortions from a highly skewed structure toward a territorially balanced configuration, while Kosovaref01 contains predominantly medium and high distortion ranges, the optimized CRS concentrates the vast majority of the territory within low-distortion intervals. Table 1 and figure 3 present the

distribution of absolute distortions for both systems. The comparison shows that the optimized CRS introduces extensive low-distortion zones that are completely absent in Kosovaref01. In particular, 65.4% of the territory falls within distortions below 50 ppm, while an additional 26.3% remains within the 50–100 ppm interval. Table 2 summarizes the principal performance indicators of both systems. The optimized projection variant increases the proportion of territory within acceptable engineering thresholds while simultaneously reducing extreme distortion values. The maximum distortion is reduced from approximately 502 ppm in Kosovaref01 to approximately 253 ppm in the optimized system.

Table 1. Distribution of absolute distortions (Kosovaref01 vs optimized CRS)

Distortion Range (cm/km)	**Ppm**	**Kosovaref01 (km²)**	**Kosovaref01 (%)**	**Optimized CRS (km²)**	**Optimized CRS (%)**
0–5	<50	0	0.0%	7123	65.4%
5–10	50–100	0	0.0%	2865	26.3%
10–15	100–150	768	7.1%	538	4.9%
15–20	150–200	3848	35.3%	253	2.3%
20–25	200–250	3789	34.8%	103	0.95%
25–30	250–300	1288	11.8%	11	0.10%
30–40	300–400	1006	9.2%	0	0%
40–50	400–500	192	1.76%	0	0%
>50	>500	2	0.02%	0	0%

Table 2. Key performance indicators

Metric	**Kosovaref01**	**Optimized CRS (k=1.00012)**
Area ≤100 ppm	**0%**	**91.7%**
Area ≤150 ppm	7.1%	98.3%
Area ≤200 ppm	42.4%	~99.6%
Area >250 ppm	22.8%	0.1%
Maximum distortion	~502 ppm	~253 ppm

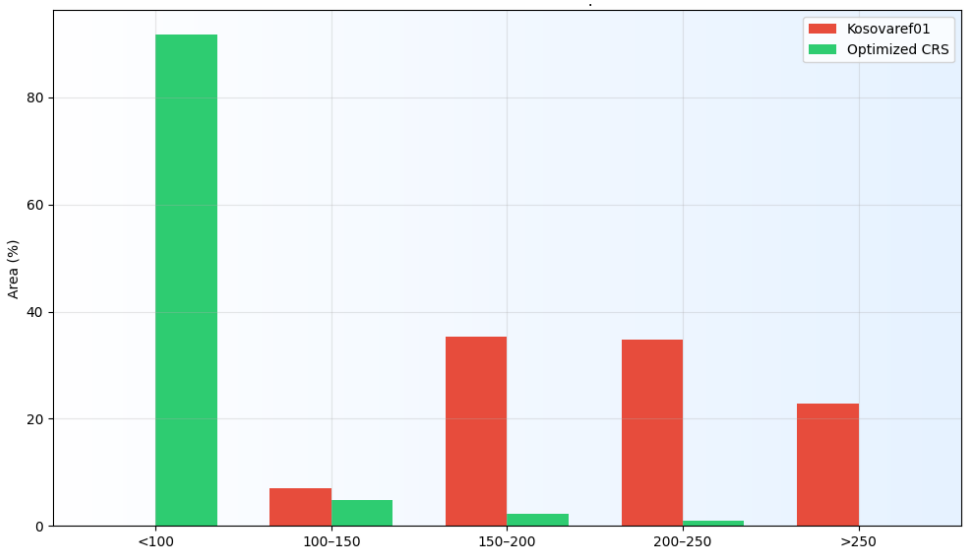


Figure 3. Distortion distribution comparison

The spatial distribution maps (Figure 4) further illustrate the difference between the two systems. In Kosovaref01, high-distortion zones dominate large portions of the territory, whereas the optimized CRS exhibits a widespread distribution of low-distortion values with only limited localized areas of higher distortions. This spatial behavior confirms that the optimization process improves both distortion magnitude and territorial balance.

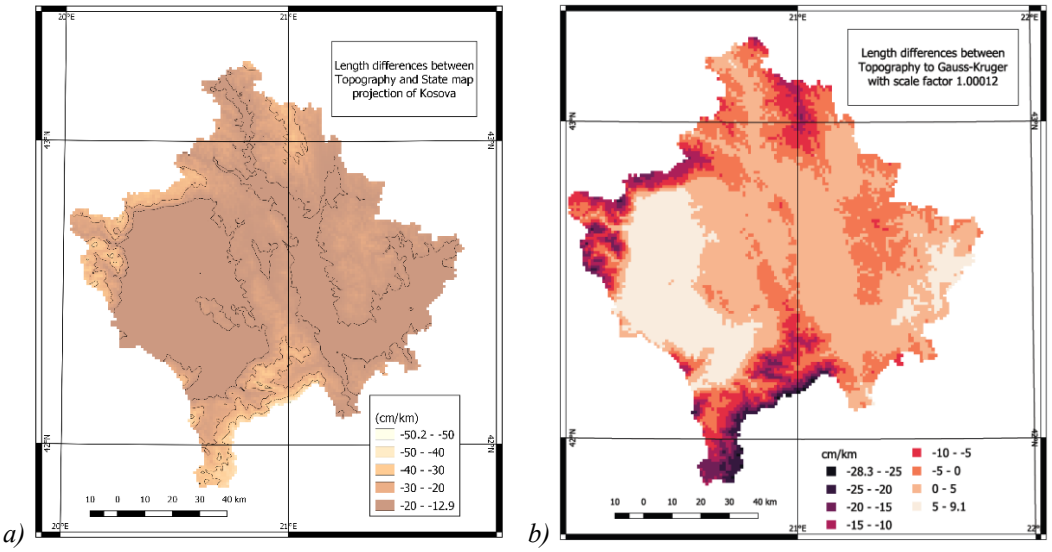


Figure 4. Maps of ground to grid distortions in a) Kosovaref01, and b) New optimized CRS with scale factor 1.00012

An additional comparison was performed between the classical ellipsoid-to-projection optimization approach and the topography-to-projection optimization methodology. The classical optimization criterion produced an optimal scale factor of 0.99996 by balancing extreme distortions on the reference ellipsoid. However, when this solution was evaluated within the integrated topography-to-projection framework, the resulting distortion structure remained entirely negative across the territory, with distortion values ranging from approximately -44.25 cm/km to -6.86 cm/km and a mean distortion of -16.13 cm/km. In contrast, the proposed optimization methodology produced a scale factor of 1.00012 and achieved a near-zero mean distortion together with a balanced distribution of positive and negative distortion zones. These results (table 3) indicate that optimization based solely on ellipsoid-to-projection relationships may remain systematically biased when evaluated against real ground-to-projection geometric conditions. It should be noted that the values reported for the optimized CRS in the comparative tables refer to the generalized scale factor k = 1.00012.

Table 3. Comparison of Classical and Topography-to-Projection Optimization Results

Metric	**Classical Optimization**	**Proposed Optimization**
Optimization basis	Ellipsoid-to-projection	Topography-to-projection
Optimal scale factor	0.99996	1.00012
Mean distortion	-16.13 cm/km	-0.14 cm/km
Minimum distortion	-6.86 cm/km	-9.14 cm/km
Maximum distortion	-44.25 cm/km	+28.26 cm/km
Positive distortions	0%	60.73%
Negative distortions	100%	39.27%
Territorial equilibrium	No	Yes

The territorial balancing effect is further illustrated in Figure 5, which presents the spatial distribution of negative ($\delta < 0$) and positive ($\delta > 0$) ground-to-grid distortions within the optimized CRS. The map demonstrates the spatial coexistence of contraction and expansion zones across the territory. It should be emphasized that the optimization criterion is not based on achieving equal territorial extents of positive and negative distortion zones, but rather on minimizing the arithmetic mean of signed distortions. Consequently, territorial equilibrium is achieved when the cumulative effects of positive and negative distortions compensate each other, causing the mean signed distortion to approach zero. As a result, areas affected by positive and negative distortions do not need to be equal in size; instead, the cumulative magnitude of their distortion effects must be approximately balanced across the territory.

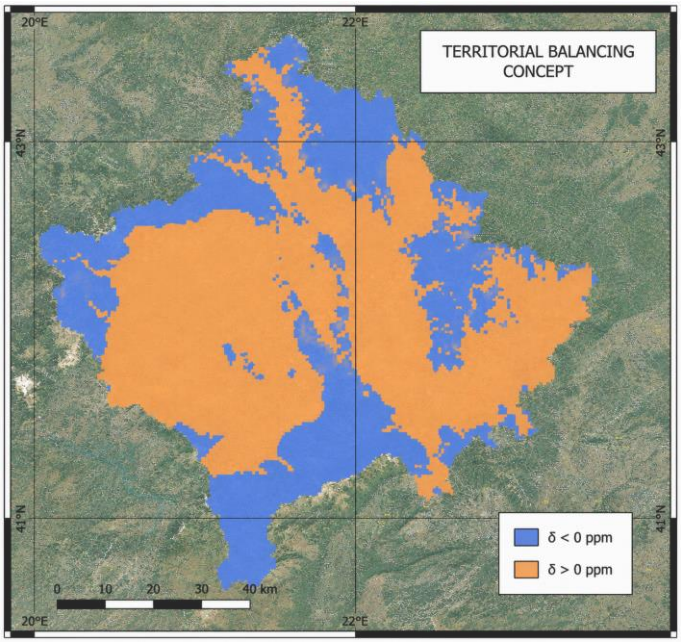


Figure 5. Territorial balancing concept for the optimized CRS (k = 1.00012), illustrating the spatial distribution of negative (δ < 0) and positive (δ > 0) ground-to-grid distortions whose cumulative effects compensate each other across the territory.

The results are additionally supported through an interactive web-based platform (figure 6), which enables visualization and inspection of distortion values at multiple spatial scales. Users can examine distortion behavior from the national level down to individual settlements, providing transparency and reproducibility of the analytical results.

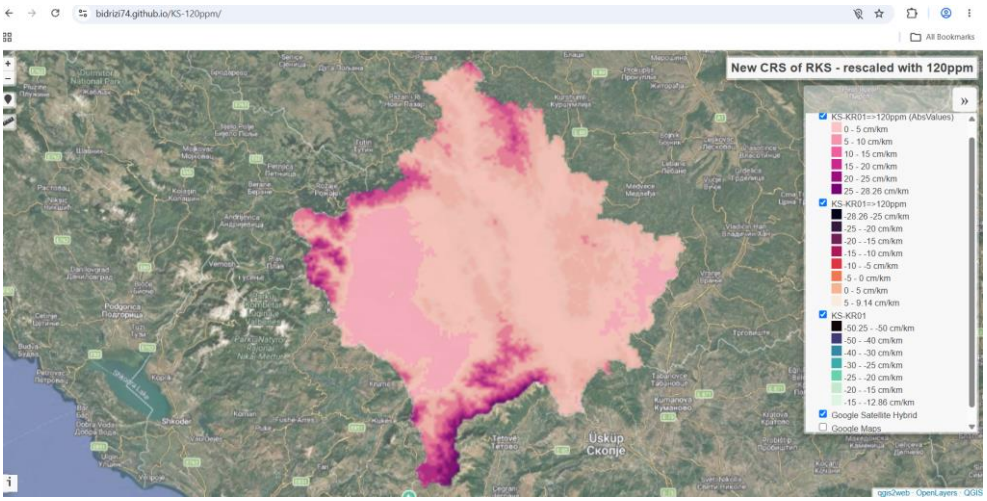


Figure 6. Interactive web-based platform for New optimized CRS of Kosova

DISCUSSION

The results of this study demonstrate that the distortion characteristics of a state-plane coordinate system are strongly dependent on the methodological framework used for projection design and optimization. The comparison between Kosovaref01 and the optimized CRS shows that projection parameters inherited from historical regional systems may generate systematic territorial distortion bias when evaluated in relation to real ground measurements (Idrizi, 2014; Dennis, 2016).

The absence of low-distortion areas below 100 ppm in Kosovaref01 (Bajrami Lubishtani et al., 2025) indicates that the current system is not geometrically balanced with respect to the physical characteristics of the territory. This behavior is directly related to the fact that the projection parameters were inherited from the former Yugoslav framework without integrated optimization based on ground-to-projection relationships.

In contrast, the optimized CRS demonstrates that the incorporation of topographic reductions and signed territorial distortion balancing significantly improves the spatial distribution of distortions. The shift of more than 90% of the territory into low-distortion ranges confirms that the proposed methodology provides a more realistic representation of spatial measurements within the projection plane.

Table 4. Interpretive Summary

Property	**Kosovaref01**	**Optimized CRS**
Distortion bias	Predominantly negative	Balanced
Distribution	Skewed	Uniform
Extreme values	High	Minimal
Suitability for engineering	Limited	High

These findings support the central premise of this study: CRS optimization should not be limited to ellipsoid-to-projection distortions (Baselga Moreno, 2020) but should instead evaluate the cumulative geometric relationship between topography and projection. Within this framework, the projection scale factor becomes a spatial optimization parameter (Baselga Moreno, 2021) whose role is to balance positive and negative distortion effects across the territory. The projection scale factor is therefore not prescribed a priori, but emerges as the optimization parameter that minimizes the mean signed ground-to-grid distortion over the territory. Because the analytical grid is composed of equal territorial units, minimization of the mean value of signed distortions simultaneously represents minimization of the cumulative territorial imbalance between positive and negative distortion zones. The effectiveness of this balancing approach is additionally confirmed by the territorial distortion equilibrium obtained in the optimized CRS variant with scale factor 1.00012. The

territorial balancing map additionally confirms that the optimization criterion is achieved not only statistically through the arithmetic mean of signed distortions, but also spatially through the near-equilibrium distribution of positive and negative distortion zones across the territory.

The optimization process converged to an exact mathematical solution (k = 1.0001213671716), producing an almost perfect territorial equilibrium with a residual imbalance of approximately 8 cm² and a mean signed distortion of only 0.0019 mm/km. Because such differences are negligible at the scale of a national coordinate reference system, the projection parameter was generalized to k=1.00012, representing a practical implementation of the mathematically optimized solution.

The results are consistent with recent developments in geodetic practice, particularly the introduction of SPCS2022 in the United States, where ground-to-grid relationships and combined scale factors have gained increasing importance (Dennis, 2023). The methodology developed in this study applies this concept through an explicitly integrated multi-surface framework incorporating topography, geoid, ellipsoid, and projection within a unified analytical model, based on research outputs from Idrizi (2014) and Idrizi et al (2018). This is particularly important because the presented methodology shifts CRS optimization from ellipsoid-based extreme-value balancing toward territorial equilibrium between real topographic space and map projection.

This study demonstrates the geometric applicability of a third optimization configuration, termed the Non-Intersecting Configuration, characterized by a projection scale factor greater than unity (k > 1) (Figure 7), which is generally absent from conventional map projection design methodologies (Idrizi 2026). The optimized solution obtained in this study is characterized by a projection scale factor greater than unity (k = 1.00012). The present results demonstrate that, when optimization is performed using topography-to-projection distortion relationships and territorial balancing criteria, positive scale-factor solutions may provide a viable alternative to the traditional tangential (k = 1) and secant (k < 1) design approaches. Although positive scale-factor configurations have previously been employed in low-distortion projection design, the present study demonstrates their effectiveness within a territorial balancing framework based on topography-to-projection distortions.

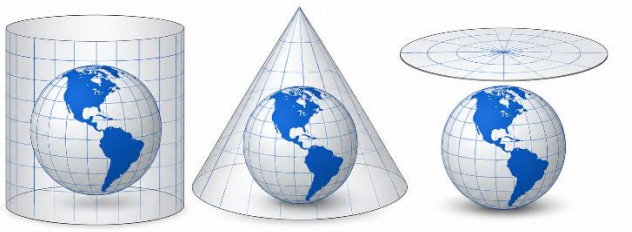


Figure 7. Non-intersecting configuration (k > 1) of cylindrical, conic, and azimuthal projection surfaces obtained by displacing the developable surface away from the reference ellipsoid (or globe)

The non-intersecting configuration used in this study should therefore be interpreted as a geometric consequence of positive scale-factor developable-surface projections rather than merely as the numerical value of the projection scale factor itself. It should be noted that positive scale factors (k > 1) are not exclusive to projections based on developable surfaces. They are also employed in projection systems derived from specific mathematical formulations that are not based on regular developable geometric surfaces. In such cases, a positive scale factor represents only a mathematical projection parameter and should not be interpreted as a non-intersecting geometric configuration. The non-intersecting interpretation proposed in this study is therefore limited to cylindrical, conic, and azimuthal projections, where the geometric relationship between the ellipsoid/globe and the developable projection surface can be explicitly defined. Consequently, the non-intersecting configuration presented in this study should be regarded as a third geometric configuration of developable-surface map projections, complementing the classical tangential and secant configurations, rather than as a general classification applicable to all map projection systems. Within this context, the non-intersecting configuration introduced in this study extends the traditional geometric classification of developable-surface map projections from two to three configurations: tangential, secant, and non-intersecting.

In the classical approach, optimization is achieved by balancing extreme distortions on the reference ellipsoid, whereas the proposed methodology seeks to balance cumulative signed distortion effects over the physical territory. Consequently, balancing extreme distortions on the reference ellipsoid does not necessarily imply balancing distortions on the physical Earth surface. The comparison presented in this study clearly demonstrates that a scale factor optimized according to classical ellipsoid-based criteria may still produce systematically biased distortion patterns when evaluated through an integrated topography-to-projection framework.

The comparison between the classical and proposed optimization approaches further highlights the conceptual difference between ellipsoid-based and topography-based CRS design. While the classical approach successfully balances extreme distortions on the reference ellipsoid and identifies an optimal scale factor of 0.99996, evaluation within the integrated topography-to-projection framework demonstrates that the resulting distortion structure remains entirely negative across the territory. In contrast, the developed methodology achieves territorial equilibrium through balancing signed distortions derived from the complete transformation chain between topography and map projection. This result demonstrates that optimization based exclusively on ellipsoid-to-projection relationships may not adequately represent real geometric conditions encountered in engineering surveying, cadastral applications, and large-scale mapping.

An important implication of this research is that CRS design can be reformulated as a territorial optimization problem rather than solely as a projection-parameter adjustment problem. The use of high-resolution spatial sampling enables direct evaluation of distortion structures across the entire territory and supports the identification of projection parameters that achieve territorial geometric equilibrium.

The proposed methodology is not limited to the case of Kosovo. The same analytical framework can be applied to other national or regional coordinate systems, particularly in territories characterized by significant topographic variation and high requirements for engineering, cadastral, and large-scale mapping accuracy. Although Kosovo was selected as the experimental case study, the optimization framework developed in this study is independent of territory and may be applied to any state-plane coordinate system where projection parameters can be adjusted and evaluated through integrated topography-to-projection analysis.

Furthermore, the methodology is not restricted to the Gauss–Krüger projection used in the present case study. Because the optimization criterion is based on topography-to-projection relationships, the same framework can be applied to any cartographic projection system whose parameters can be iteratively adjusted and evaluated through signed ground-to-grid distortions.

In some territories, optimization based on the entire national area may not necessarily provide the most practical solution. Regions characterized by extensive mountainous areas with limited engineering, cadastral, or infrastructure activity may influence the optimization process disproportionately relative to their practical significance. Another direction for future research will focus on the development of Selective Territorial Optimization (STO), in which the optimization process is performed over the entire national coordinate reference system while the territorial balancing criterion is applied only to predefined areas of strategic or practical importance. Such an approach is expected to provide higher geometric consistency within engineering, cadastral, transportation, urban, or other priority regions without affecting the integrity of the national coordinate reference system.

The optimization framework presented in this study is not restricted to the territorial balancing criterion adopted herein. More generally, the proposed iterative optimization model provides a flexible mathematical framework in which different optimization objectives may be implemented by modifying the objective function or optimization criterion while preserving the same computational procedure. In the present study, the objective is to minimize the arithmetic mean of signed topography-to-projection distortions, thereby achieving territorial equilibrium between cumulative positive and negative distortion effects. However, alternative optimization criteria may also be incorporated depending on the intended application of the state-plane coordinate system. Such criteria could include optimization focused on specific regions of interest, engineering corridors, cadastral zones, urban areas, or other application-dependent spatial requirements. Consequently, the presented methodology should be regarded as a general iterative optimization framework, of which territorial balancing represents one particular application.

The obtained optimization results are influenced by the adopted analytical resolution (1 km × 1 km grid) and by the accuracy of the elevation and geoid models used in the analysis. Different spatial resolutions or improved geodetic models may produce slightly different optimal scale factors, although the general optimization principle remains unchanged.

Furthermore, the integration of an interactive web-based platform (figure 6) introduces an additional level of transparency and reproducibility, enabling direct visualization and verification of distortion behavior at local spatial scales.

The applicability of the presented methodology depends on the relationship between topography-to-projection distortions and the positional accuracy required by the intended geospatial product (figure 8). When distortions introduced by topography and geoid reductions are negligible relative to the mapping accuracy determined by the map scale and application requirements, the conventional ellipsoid-to-projection optimization approach remains appropriate. Conversely, in engineering geodesy, cadastral systems, large-scale topographic mapping, and other high-precision applications, where ground-to-grid consistency becomes critical, the integrated topography-to-projection optimization framework provides a more realistic basis for coordinate system design.

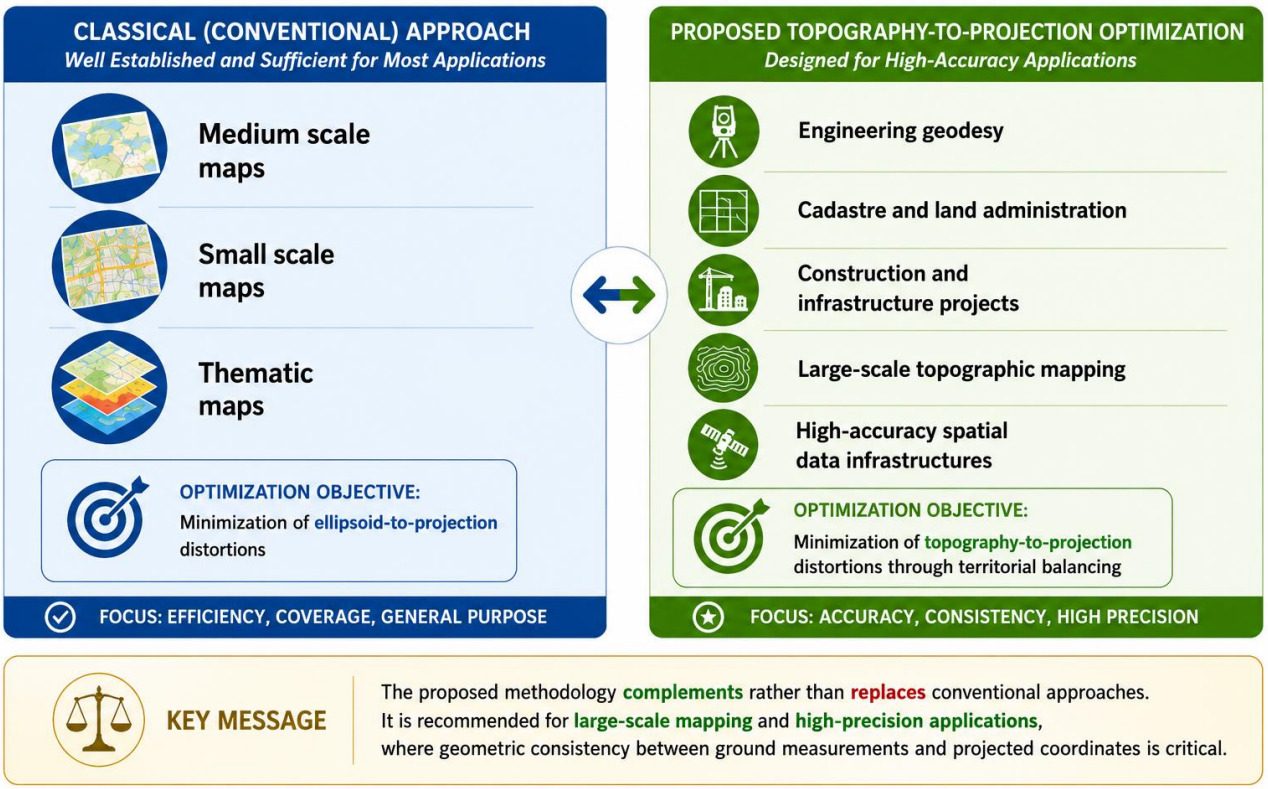


Figure 8. Application domains of classical and topography-to-projection optimization approaches

CONCLUSION

The principal scientific contributions of this study can be summarized as follows:

1. **A generalized topography-to-projection optimization framework**, replacing conventional ellipsoid-to-projection optimization with an integrated multi-surface model;
2. **A territorial balancing optimization criterion**, based on minimizing the arithmetic mean of signed ground-to-grid distortions rather than balancing extreme ellipsoidal distortions;
3. **A generalized iterative optimization framework**, capable of supporting alternative optimization criteria through modification of the objective function while preserving the computational procedure; and
4. **A geometric interpretation of positive scale-factor (k > 1) developable-surface projections** through the proposed non-intersecting configuration.

This study presents a methodological framework for the optimization of state-plane coordinate systems through integrated analysis of geometric relationships between topography and map projection. Unlike conventional ellipsoid to map projection-based approaches, the proposed methodology evaluates distortions as cumulative ground-to-grid differences across multiple reference surfaces, including topography, geoid, ellipsoid, and projection.

The application of the multi-surface framework to the Republic of Kosovo demonstrates that the current CRS, Kosovaref01, exhibits a systematically unbalanced distortion structure inherited from historical projection parameters that were not optimized with respect to ground-to-projection relationships. Through iterative adjustment of the projection scale factor and evaluation of signed territorial distortions, an optimized projection variant was developed.

The optimized CRS substantially improves distortion characteristics across the national territory. More than 90% of the territory falls within distortion ranges below 100 ppm, while extreme distortions are significantly reduced compared to the existing system. The optimization process also produces a territorially balanced distortion structure in which positive and negative distortion effects compensate spatially across the analytical grid. The optimized framework also enables a

territorially normalized representation of the Republic of Kosovo, resulting in an adjusted territorial area after integrated topography-to-projection reduction.

The results confirm that CRS optimization should not be restricted to ellipsoid-to-projection distortions alone, but should instead incorporate the complete geometric transformation chain from topography to map projection. Within this framework, the projection scale factor functions as a territorial balancing parameter that minimizes systematic geometric bias between real-world measurements and projected cartesian coordinates. In contrast to conventional ellipsoid-based projection optimization, the presented framework introduces territorial balancing of signed topography-to-projection distortions as the principal optimization criterion. The optimization converged to an exact mathematical solution (k=1.0001213671716), yielding a mean signed distortion of only 0.0019 mm/km (0.0019 ppm), while the remaining territorial imbalance corresponds to approximately 8 cm² over the entire territory. For practical implementation, the scale factor was generalized to k = 1.00012 because the difference is scientifically and practically negligible for nationwide coordinate system design.

It should be emphasized that the optimization presented in this study was performed within the existing Gauss–Krüger projection framework currently used in Kosovo. Consequently, the objective of the research is not to determine the single definitive projection solution for the territory, but rather to establish a methodological framework for CRS optimization applicable to different map projection systems and territorial configurations. Future research may therefore include the evaluation and optimization of alternative map projection types in order to identify potentially more suitable projection variants under the presented topography-to-projection optimization framework. This represents one possible extension of the methodology developed in this study.

Beyond the case of Kosovo, the proposed methodology provides a generalizable framework for the design and optimization of modern state-plane coordinate systems, particularly for engineering geodesy, cadastral systems, and large-scale mapping applications requiring high geometric consistency between ground measurements and projected spatial data. The Republic of Kosovo serves solely as a validation case study for the proposed methodology, whereas the optimization framework is intended to be applicable to different territories, map projection systems, and coordinate system design problems. The framework developed in this study is particularly suitable for engineering geodesy, cadastral systems, and large-scale mapping, where high geometric consistency between ground measurements and projected coordinates is essential. For medium-scale, small-scale, and thematic mapping, however, conventional projection design methodologies generally remain appropriate, as their geometric accuracy requirements differ substantially from those of high-precision geospatial applications.

Beyond the territorial balancing criterion demonstrated in this study, the iterative optimization framework established here provides a general methodology for optimizing state-plane coordinate systems under alternative application-specific optimization criteria.

The proposed methodology is therefore not intended to replace conventional projection optimization universally, but to complement it in situations where topography-to-projection distortions become significant relative to the positional accuracy required by the intended mapping or engineering application.

The present study establishes the theoretical and mathematical basis for this subsequent research direction. An important direction for future research will be the development of a Selective Territorial Optimization (STO), in which the optimization process is performed over the entire national coordinate system while the balancing criterion is applied only to predefined areas of practical importance. Such areas may include engineering corridors, cadastral regions, urban zones, transportation networks, or other application-specific territories. This approach is expected to further improve geometric consistency in areas where high-accuracy geospatial applications are concentrated, while preserving a unified state plane coordinate system, as well as local plane coordinate systems.

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BIOGRAPHY



Prof.Dr. Bashkim IDRIZI, was born on 14.07.1974 in Skopje, Macedonia. He graduated in geodesy department of the Polytechnic University of Tirana-Albania in 1999. In 2004 he got the master degree of sciences (MSc) in Ss.Cyril and Methodius University-Skopje. In 2005 he had a specialization for Global Mapping in Geographical-Survey Institute (GSI) of Japan in Tsukuba-Japan. In 2007 he held the degree of Doctor of Sciences (PhD) in Geodesy department of Ss.Cyril and Methodius University–Skopje. He worked in State Authority for Geodetic Works from May 1999 until January 2008. During this time he appointed for head of cartography department in 2004, i.e. geodetic works. From October 2003 up to January 2008, he worked as a outsourcing lecturer in State University of Tetova, while from 2005 to 2018 he worked as part time lecturer in the University of Prishtina. From February 2008 to October 2016 he worked as a cartography and GIS Professor at the State University of Tetova–Tetova. From November 2016 to November 2018 he worked as cartography and GIS lecturer at the University "Mother Teresa" in Skopje. From December 2018, he is Full Professor in geodesy department of the University of Prishtina - Kosova. He is the author of six cartography university books, and more than 180 papers published and presented in national and international scientific conferences related to geodesy, cartography, GIS and remote sensing. From March 2010, he is appointed as president of Geo-SEE (South-European Research Institute on Geo Sciences). On November 2011, he has been elected as first President of Pan-national Association of Albanian Surveyors "Karl Gega", and his mandate ended on February 2014.