

ARTIFICIAL INTELLIGENCE TOOL AND GEOGRAPHICAL INFORMATION TO IMPLANT NETWORKS

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Abstract

In France, we can list more than 4 million kilometers of networks (buried or aeriean ones) and around 100 000 network accidents during construction works. Some important accidents involving networks and especially buried ones happen all around the world (Table 1).

Table1: Examples of pipe accidents

<i>Town</i>	<i>Date</i>	<i>Causes</i>	<i>Damages</i>
<i>Dijon (France)</i>	<i>12/1999</i>	<i>Break of a gas pipe</i>	<i>11 people died 3 seriously injured</i>
<i>Ghislenghien (Belgium)</i>	<i>07/2004</i>	<i>Damaged pipe because of previous work</i>	<i>25 people died 150 seriously injured</i>
<i>Beijing (China)</i>	<i>11/2013</i>	<i>Rupture of an oil pipe</i>	<i>63 people died 150 injured</i>
<i>East Harlem (USA)</i>	<i>03/2014</i>	<i>Gas leak</i>	<i>8 people died 70 injured</i>
<i>Ludwigshafen (Germany)</i>	<i>10/2014</i>	<i>Excavation in the vicinity of a gas pipe</i>	<i>1 people died 10 injured</i>

To prevent such accidents, new french regulations enter into force. The assistance for construction contractors to implant some new pipes is an interesting and appropriate research. This study deals with the creation of a software able to link the regulations and the geographical network information taking all their data into account (such as coordinate precision, security area, topological relations and so on). This software should be a tool for decision support on the borderline between a Geographical Information System and an Artificial Intelligence tool.

Keywords: *Artificial Intelligence, Expert System, Network Regulations, Pipe Damages, Network Security*

INTRODUCTION

Disasters like the ones that happened in Ghislenghien (Belgium), Ludwigshafen (Germany), or Lyon (France), have been attributed to an excavation in the vicinity of gas pipelines. Leaks can also happen as a result of an ageing gas network whose pipelines may corrode and rupture (by example in East Harlem, in U.S.A., and in Dijon, in France) or climate event, as it was the case in the destruction of New Orleans gas network after Hurricane Katrina in 2005. Such tragic occurrences push to increase the security level. A preventive strategy must be adopted to reduce them. That's why it is necessary to create a software able to verify the good implementation of the networks (Figure 1) taking network geographical data into account.



Figure 1: *Network implementations (cnig.gouv.fr)*

OBJECTIVES

Pipe implementations have to meet several security device criteria to prevent accidents. For example, sanitation networks have a minimal inclination in order to ensure a good rate of flow and the water pipe depth must be upper than the frost one. For the dangerous ones, as it is the case with high pressure gas pipelines or petroleum ones, it is necessary to acquire a large security area.

The French current network regulations (Multifluide and DT/DICT) are explained in the next section. Thanks to these new regulations and the network geographical data, the developed software must :

- determine the best implementation rules for a network,
- control if a buried network respects current regulations according to its characteristics,
- inform about the expected security device implementations.

Such a study could ensure a best knowledge about the networks and a security improvement.

WHY ARE THE URBAN NETWORKS MODELIZED?

Network accidents

The existing networks, the labyrinth of pipelines, big and small, which transport electricity, water, gas..., from wellhead to home, are important and getting more important. If pipes are one of the safest methods of transportation for conveying hazardous substances, the loss of containment following a pipeline fracture or accidental release could have disastrous consequences.



Figure 2: Picture of the Ghislenghien's explosion (From <http://www.lessentiel.lu/news/story/28204100>)

In the following table, some of the most important pipes accidents are reported:

Table2: Examples of pipes accidents

Town	Date	Causes
Ghislenghien (Belgium)	July 2004	Damaged pipe because of previous work
Bondy (France)	November 2007	Unlisted pipe
Lyon (France)	February 2008	Works near a gas pipe
Plaine de la Crau (France)	August 2009	Break of a hydrocarbon pipe
Sablé sur Sarthe (France)	July 2012	A mechanical digger damaged a gas pipe
East Harlem(USA)	March 2014	Gas leak

That's why it is necessary to acquire and control the 3D information on the different city networks, especially for buried ones in connection with the modernization, maintenance, inspection and elimination of accidents, as well as for access to their location.

The Buried Network Regulations

This work is based on the French regulations about buried networks. They are called "Arrêté Multifluide" and "Réforme DT/DICT". The "Arrêté Multifluide" deals with the security characteristics to establish security devices concerning the dangerous networks (such as high pressure gas pipelines), and the "Reforme DT/DICT" is referring to the security area around a network and, more especially, when its coordinates have an important uncertainty. These regulations represent the basis of the rules-data-set of the Expert System developed in this Software.

HOW ARE THE NETWORKS MODELIZED?

The network representation as a graph

Due to the existence of a multitude of underground networks that overlap or intersect, it is important to identify, collect information and study the uncertainties and the topological relations of each network in the vicinity of the other pipes. This paper deals with the characterization and the structure of the relations between the networks and their representation.

The 3D nature of cities causes data to mount up. On one geographic spot the data may record different networks. That's the reason why we focus on the representation of geomatic applications. One of the usual representations is the graph. According to the theory, a graph is composed of Vertices and Edges with pair wise relations. Thanks to a succession of Arcs, each of them being defined by two Vertices, a network can be considered as a graph. Each Arc supports Auxiliary Points (cutting this Arc in Segments). In the same vein, we may add some Furniture (such as manholes for a water network, or street lighting poles for the electric one) between these Auxiliary Points, cutting the Segments in Sections (Figure 3).

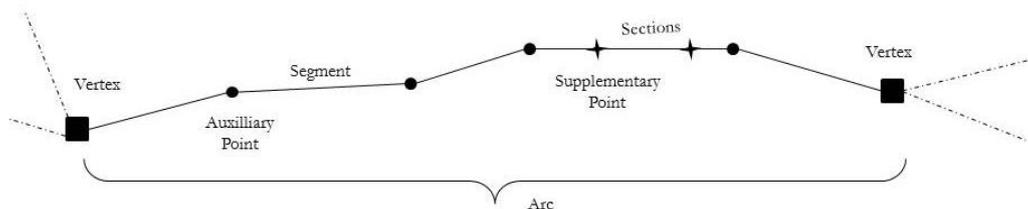


Figure 3: Graph representation

Another representation of the graph structure is possible with the HBDS^[1] one (Figure 4). This method (Hypergraph-Based Data Structure) gives the data structure from the phenomena structure. Each element can be represented as a class, an object, an attribute or a link, such as an arc which is an object of the arc class, and so on. In this study, each class depicts an element of the graph (Arc, Section, Vertex, Auxiliary Point -which belongs to the arc but isn't a Vertex, ...). The HBDS structure enables the illustration of the relations between these classes (for instance, Arc "is composed of" Vertices, Segment "supports" Supplementary Points, ...).

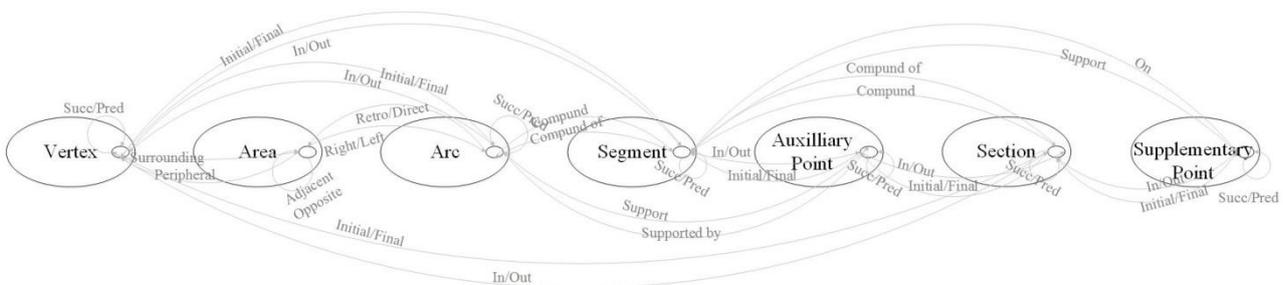


Figure 4: HBDS Graph organization

The network uncertainties

The buried pipelines can be depicted, without considering the vagueness or with blur (cf. next figure).

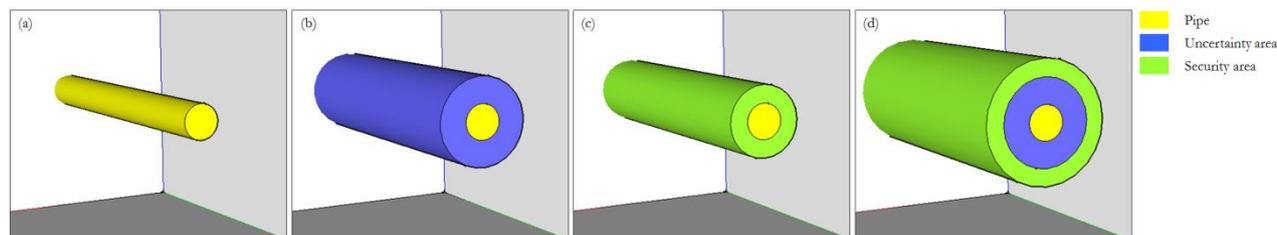


Figure 5: Pipes representation: (a) without blur, (b) with coordinates uncertainty, (c) within the security area, (d) pipe within security area and the uncertainty area

AN ARTIFICIAL INTELLIGENCE TOOL ABLE TO GIVE THE NETWORK IMPLANTATION CHARACTERISTICS

Why an Artificial Intelligence tool?

The implantation of a new network involves taking care of the environment and more particularly the buried one. That's why the modelization of the underground is necessary. The French regulations engage in identifying the position of the underground networks before realizing some roadworks. That's why we often see some marks on the pavements and roads to show the network positions (Figure 6).



Figure 6: Photography of network position marks in Paris

Taking the network geographical data into account requires to have a tool able to generate new information from an important data base. This tool needs a large range of data and the current regulations to provide the users the most precise answers to implant a new network in a very occupied underground. An Expert System can consider a lot of data to give the administratives or the concessionaries an answer thanks to a rule base linked to the regulations. That's why the creation of an Artificial Intelligence tool seems to be the right choice.

General Principle

The development of an Artificial Intelligence tool contributes to the challenge providing the users and consumers a unique tool set able to:

- determine the fluid classes,
- define the kinds of pipes,
- inform about the required standards associated with the networks characteristics,
- assess level accuracy and so define the risks of damage.

These various forms of data contribute to the challenge of running and achieving the programs, according to the user's requests.

This Expert System is composed of a Fact base (on the user's knowledge), a Rule base (determined by the current regulations) and a main function able to link these elements.

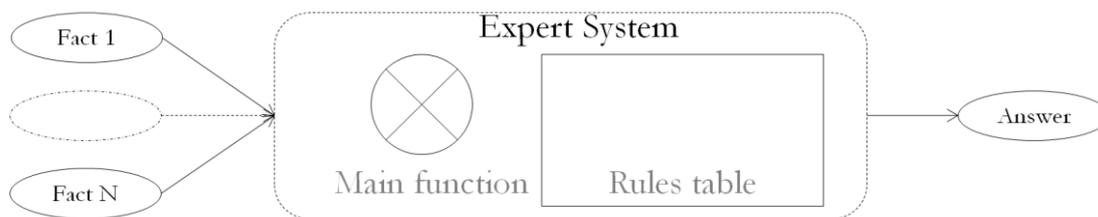


Figure 7: Expert System structure

Some rule examples are shown in this paragraph. In an Expert System^[6, 7], a rule is an expression composed by facts connected with some logical operators (And, Or,...) and can concern:

- materials for a kind of pipe
 - IF the pipe is for telecommunication
 - SO the warning materials must be green
- building rules
 - IF the pipe is carrying gas
 - SO the minimal depth is 0,30m
- security rules verifying whether two networks are impacting each other or not
 - IF a plumb pipe is carrying gas
 - AND a pipe is carrying warm water in the surroundings
 - AND both pipes are parallel
 - SO the distance between the pipes must be greater than or equal to 3m

Algorithm 1: The Expert System Algorithm

```

∇
RuleValue 0 ≤
    TYPE == gas ?
        SECURITYDISTANCE ← 0,20m (except if warm water is in the vicinity) ;
        WARNINGCOLOR ← Yellow; ...
    |'
    TYPE == electricity ?
        WARNINGCOLOR ← Red; ...
    |'
    ...
    !
/
RuleValue 1 ≤
    TYPE == gas && MODE == distribution ?
        WARNINGMATERIAL ← marker || flush ...
    |'
    ...
    !
/...
∞
    !
    
```

RESULTS

Who could be interested in?

A lot of trades are implied when a new pipe is on working: network concessionaries, urban and rural districts, construction contractors, public area managers, and so on. In fact, to implant a new network, a lot of information are required, such as :

- the kind of pipe
- the type of transported fluid,
- the implantation area data (the other pipes in the vicinity, the soil characteristics, and so on),
- the situation of the yard and its hold on the ground,
- the current regulation about network security (minimal distance between two pipes, coordinate uncertainty determination, and so on).

Risk cartography examples

The implantation of a new network requires to verify if the network characteristics are in line with the current French regulations:

- the specific characteristics of the future network (such as security device color, minimal diameter,...),
- the minimal security distance with the other pipes in its vicinity,
- the minimal depth proposed in the regulations.

What should be the network characteristics?

The rule base of the Expert Sytem is linked to the current French regulations, which give a lot of implant characteristic obligations. To get these informations, the software requires some data such as the kind of pipe, the requested data,... Some requests are simple and need only one data. By example, the user needs to know which is the color of the security device for a gaz pipe, or the minimal distance with the vegetation for an electric one:

<pre>Fact1: TYPE Fact1 value: GAS Request: securitydevicecolor Transit by rule 0 Answer: yellow</pre>	<pre>Fact1: TYPE Fact1 value: Electricity Request: vegetationdistance Transit by rule 0 Answer: 1.5</pre>
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Figure 8: Examples of simple requests

And some others are more intricate, as it is the case for the minimal slope determination where the type of pipe (sewerage by example) and the transported fluid (rainy waters) are needed;

```
Fact1: TYPE
Fact1 value: Sewage
Fact2: burriedfluid
Fact2 value: rainwater
Request: minimalslop
Transit by rule 0
Transit by rule 2
Transit by rule 3
Transit by rule 4
Transit by rule 5
Transit by rule 6
Transit by rule 7
Transit by rule 8
Answer: 0.004%
```

Figure 9: Example of a complex request

Is the new network affecting the other ones in the surroundings?

The French regulations indicate:

- the minimal security distance between the studied network and the other ones,
- the coordinate uncertainties.

The calculation of the minimal security distance between two networks takes the maximal security radius of these pipes into account. By example, if the security radius of an electric network is 20cm and the security distance of an internet pipe is 5cm, the minimal security distance between these two pipes is at least 20cm. This request can be illustrated on a cartography where the networks affected by the new one are colored (in blue).

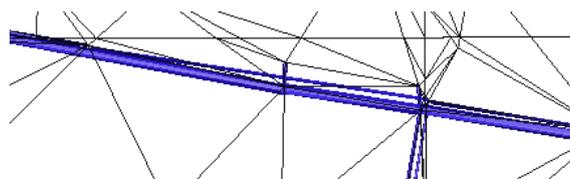


Figure 10: Networks affected by other ones (in blue)

Is the new network depth enough?

This new step is considering a Digital Elevating Model. Indeed this calculation is based on the minimal distance between the studied pipe and the floor to know if the new implantation verifies the regulations. An other interest for this determination is to confirm before working, if the future trench will not affect the implanted pipes. This can be represented by two methods: the first one colores the affected network(s) in red, the second one colores the risk zone(s) with a color gradient (in green the non affected area, in dark red the most affected one).

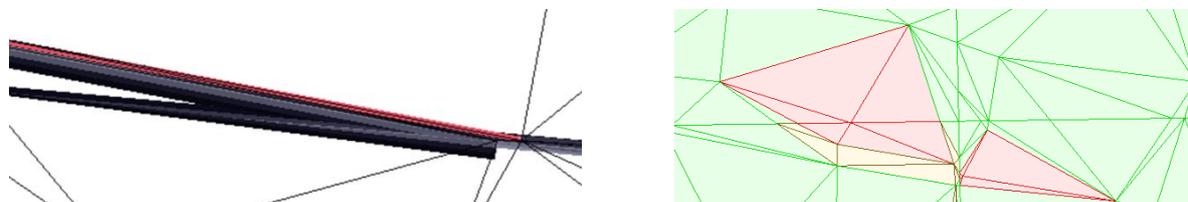


Figure 11: Networks affected by the soil (in red) and affected network area (in red shades)

All these representation cases can be independent or on the same cartography:

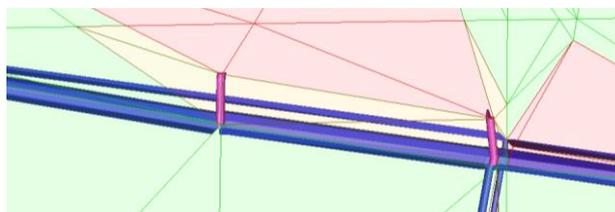


Figure 12: A cartography with all the determined information.

CONCLUSION

Underground is encumbered. Managing all the risks of pipe damages while working remains difficult, that's why informatic gives administratives and concessionaries new tools to prevent accidents, to verify the current regulations, to work with invisible data. Such software should help the decision for the implementation of a new pipe or for other works on pipes. This kind of tool, at the crossroads of Geographical Data and Artificial Intelligence, presents a lot of advantages, giving the users a risk map, using a very large panel of data to establish the risk determination, taking current regulations into account, and so on. However, it needs the most accurate data as possible to obtain the best

results and to deliver the best information. It is primarily geared to all the trades which work on networks. That's the reason why this tool must be easy to be used and to be updated according to the future modifications.

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ANNEXES

Mathematical methods

The minimal inclination compliance - Le respect des pentes

Several networks categories have, in their regulation characteristics, a minimal inclination to respect. Thanks to 3D network data, this software is able to determine the inclination of a network taking the geographical coordinates of two points. The next algorithm deals with this problematic.

Algorithm 2: The inclination calculation

$\text{dist} \leftarrow \sqrt{((XB-XA)*(XB-XA) + (YB-YA)*(YB-YA))}$ $\text{heightDifference} \leftarrow \text{abs}(ZB-ZA);$ $\text{inclination} \leftarrow \text{dist}/\text{heightDifference};$
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The minimal depth - distance calculation with the Digital Elevating Model

In order to make contact calculations easier, we schematize pipelines as regular cylinders and the Digital Elevating Model as a set of triangles^[5]. This step consists in studying the minimal distance between two 3D segments (Euclidian distance) or between a 3D point and a 3D segment out (Figure 13).

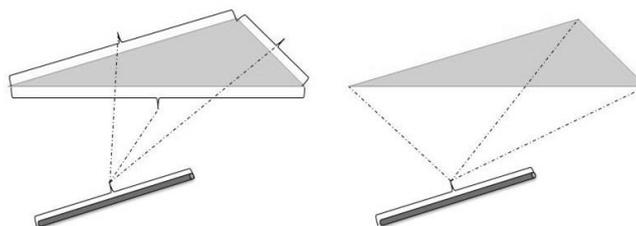


Figure 13: Minimal distance calculation between a DEM triangle and a pipe

According to these distance calculations and to the uncertainty characterizations, we define the risks of intersection between the networks and the soil surface. Taking the blur into account, we'll study the accuracy of the data and the approximate calculations directly related to significant numbers.

To evaluate the distance, the main calculations are:

Algorithm 3: The distance calculation between a segment and a point

$X_u \otimes X_B - X_A ; Y_u \otimes Y_B - Y_A ;$ $Z_u \otimes Z_B - Z_A ;$ $D \otimes -1 * (X_u * X_M + Y_u * Y_M + Z_u * Z_M) ;$ $k \otimes (-1 * D - X_B * X_u - Y_B * Y_u - Z_B * Z_u) / (X_u * X_u + Y_u * Y_u + Z_u * Z_u) ;$ $X_H \otimes X_u * k + X_B ;$ $Y_H \otimes Y_u * k + Y_B ;$ $Z_H \otimes Z_u * k + Z_B ;$ $X_H > \min(X_A, X_B) \geq X_H < \max(X_A, X_B) \geq Y_H > \min(Y_A, Y_B) \geq Y_H < \max(Y_A, Y_B) \wedge$ $\mathcal{R} \text{In the segment } \otimes \Upsilon$ $d_1 \otimes \uparrow \left((X_A - X_H)^2 + (Y_A - Y_H)^2 \right) ;$	$d_2 \otimes \uparrow \left((X_B - X_H)^2 + (Y_B - Y_H)^2 \right) ;$ $d_1 < d_2 \wedge$ $X_H \otimes X_A ;$ $Y_H \otimes Y_A ;$ $Z_H \otimes Z_A ;$ Υ $X_H \otimes X_B ;$ $Y_H \otimes Y_B ;$ $Z_H \otimes Z_B ;$ $\text{dist} \leftarrow \uparrow \left((X_H - X_M)^2 + (Y_H - Y_M)^2 + (Z_H - Z_M)^2 \right) ;$
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Algorithm 4: The distance calculation between two segments

$X_{S1} \otimes S1X2 - S1X1 ; Y_{S1} \otimes S1Y2 - S1Y1 ; Z_{S1} \otimes S1Z2 - S1Z1$ $\text{NormeS1} \otimes \text{sqrt}(X_{S1}^2 + Y_{S1}^2 + Z_{S1}^2)$ $Y_{S2} \otimes S2Y2 - S2Y1$ $\text{coefa} \otimes Y_{S1} / X_{S1} ; \text{coefb} \otimes S1Y1 - \text{coefa} * S1X1 ;$ $Y_{th} \otimes \text{coefa} * S2X1 + \text{coefb} ; Y_{th2} \otimes \text{coefa} * S2X2 + \text{coefb}$ $X_{S2} \otimes S2X2 - S2X1 ; Y_{S2} \otimes S2Y2 - S2Y1 ; Z_{S2} \otimes S2Z2 - S2Z1$ $\text{NormeS2} \otimes \text{sqrt}(X_{S2}^2 + Y_{S2}^2 + Z_{S2}^2)$ $X_{S1} \text{scalS2} \otimes Y_{S1} * Z_{S2} - Y_{S2} * Z_{S1} ; Y_{S1} \text{scalS2} \otimes Z_{S1} * X_{S2} - Z_{S2} * X_{S1}$ $Z_{S1} \text{scalS2} \otimes X_{S1} * Y_{S2} - X_{S2} * Y_{S1}$ $\text{NormeS1scalS2} \otimes \text{sqrt}(X_{S1} \text{scalS2}^2 + Y_{S1} \text{scalS2}^2 + Z_{S1} \text{scalS2}^2)$ $b_1 \otimes \min(S1Y1, S1Y2) \leq S2Y1 \leq \max(S1Y1, S1Y2) \mid \mid \min(S1X1, S1X2) \leq S2X1 \leq \max(S1X1, S1X2) ;$ $b_2 \otimes \min(S1Y1, S1Y2) \leq S2Y2 \leq \max(S1Y1, S1Y2) \mid \mid \min(S1X1, S1X2) \leq S2X2 \leq \max(S1X1, S1X2) ;$ $b_3 \otimes \min(S2Y1, S2Y2) \leq S1Y1 \leq \max(S2Y1, S2Y2) \mid \mid \min(S2X1, S2X2) \leq S1X1 \leq \max(S2X1, S2X2) ;$ $b_4 \otimes \min(S2Y1, S2Y2) \leq S1Y2 \leq \max(S2Y1, S2Y2) \mid \mid \min(S2X1, S2X2) \leq S1X2 \leq \max(S2X1, S2X2) ;$ $ba \otimes \min(S2X1, S2X2) \leq \max(S1X1, S1X2)$ $bb \otimes \min(S2Y1, S2Y2) \leq \max(S1Y1, S1Y2)$ $bc \otimes \min(S1X1, S1X2) \leq \max(S2X1, S2X2)$ $bd \otimes \min(S1Y1, S1Y2) \leq \max(S2Y1, S2Y2)$ $\text{NormeS1scalS2} \neq 0 \ \&\& \ (ba \ \&\& \ bb \ \&\& \ bc \ \&\& \ bd \ \&\& \ ((S2Y1 > Y_{th} \ \&\& \ S2Y2 < Y_{th2}) \mid \mid (S2Y1 < Y_{th} \ \&\& \ S2Y2 > Y_{th2}))) \wedge$ $\mathcal{R} \text{distance determination between two intersecting segments } \otimes$ $X_{S1S2} \otimes S2X2 - S1X1 ; Y_{S1S2} \otimes S2Y1 - S1Y1 ; Z_{S1S2} \otimes S2Z1 - S1Z1$ $\text{det}_{S1S2} \otimes X_{S1S2} * Y_{S1} * Z_{S2} + X_{S1} * Y_{S2} * Z_{S1S2} + X_{S2} * Y_{S1S2} * Z_{S1} - X_{S2} * Y_{S1} * Z_{S1S2} - Y_{S2} * Z_{S1} * X_{S1S2} - Z_{S2} * Y_{S1S2} * X_{S1}$ $\text{DistA} \otimes \text{abs}(\text{det}_{S1S2}) / \text{NormeS1scalS2}$ Υ $(ba \Rightarrow \&\& \ bc \Rightarrow \&\& \ (bb \Rightarrow _ \mid \mid \ bd \Rightarrow _)) \mid \mid$ $(bb \Rightarrow \&\& \ \&\& \ \&\& \ (ba \Rightarrow _ \mid \mid \ bc \Rightarrow _)) \wedge$ $\text{NormeS1} \neq 0 \wedge$ $\mathcal{R} \text{Taking into account the D2 points and the segment S1 } \otimes$ $X_{S2_1_S1_1} \otimes S2X1 - S1X1 ; Y_{S2_1_S1_1} \otimes S2Y1 - S1Y1$ $Z_{S2_1_S1_1} \otimes S2Z1 - S1Z1$ $X_{S1} \text{scalS2_1_S1_1} \otimes Y_{S1} * Z_{S2_1_S1_1} - Y_{S2_1_S1_1} * Z_{S1}$	$X_{S2_2_S1_1} \otimes S2X2 - S1X1 ; Y_{S2_2_S1_1} \otimes S2Y2 - S1Y1$ $Z_{S2_2_S1_1} \otimes S2Z2 - S1Z1$ $X_{S1} \text{scalS2_2_S1_1} \otimes Y_{S1} * Z_{S2_2_S1_1} - Y_{S2_2_S1_1} * Z_{S1}$ $Y_{S1} \text{scalS2_2_S1_1} \otimes Z_{S1} * X_{S2_2_S1_1} - Z_{S2_2_S1_1} * X_{S1}$ $Z_{S1} \text{scalS2_2_S1_1} \otimes X_{S1} * Y_{S2_2_S1_1} - X_{S2_2_S1_1} * Y_{S1}$ $\text{NormeS1scalS2_2_S1_1} \otimes \text{sqrt}(X_{S1} \text{scalS2_2_S1_1}^2 + Y_{S1} \text{scalS2_2_S1_1}^2 + Z_{S1} \text{scalS2_2_S1_1}^2)$ $\text{DistC} \otimes \text{NormeS1scalS2_2_S1_1} / \text{NormeS1}$ Υ' $\text{NormeS2} \neq 0 \wedge$ $\mathcal{R} \text{Taking into account the D1 points and the segment S2 } \otimes$ $X_{S1_1_S2_1} \otimes S2X1 - S1X1 ; Y_{S1_1_S2_1} \otimes S2Y1 - S1Y1$ $Z_{S1_1_S2_1} \otimes S2Z1 - S1Z1$ $X_{S1} \text{scalS1_1_S2_1} \otimes Y_{S2} * Z_{S1_1_S2_1} - Y_{S1_1_S2_1} * Z_{S2}$ $Y_{S1} \text{scalS1_1_S2_1} \otimes Z_{S2} * X_{S1_1_S2_1} - Z_{S1_1_S2_1} * X_{S2}$ $Z_{S1} \text{scalS1_1_S2_1} \otimes X_{S2} * Y_{S1_1_S2_1} - X_{S1_1_S2_1} * Y_{S2}$ $\text{NormeS1scalS1_1_S2_1} \otimes \text{sqrt}(X_{S1} \text{scalS1_1_S2_1}^2 + Y_{S1} \text{scalS1_1_S2_1}^2 + Z_{S1} \text{scalS1_1_S2_1}^2)$ $\text{DistD} \otimes \text{NormeS1scalS1_1_S2_1} / \text{NormeS2}$ $X_{S1_2_S2_1} \otimes S2X2 - S1X1 ; Y_{S1_2_S2_1} \otimes S2Y2 - S1Y1$ $Z_{S1_2_S2_1} \otimes S2Z2 - S1Z1$ $X_{S1} \text{scalS2_2_S1_1} \otimes Y_{S2} * Z_{S1_2_S2_1} - Y_{S1_2_S2_1} * Z_{S2}$ $Y_{S1} \text{scalS2_2_S1_1} \otimes Z_{S2} * X_{S1_2_S2_1} - Z_{S1_2_S2_1} * X_{S2}$ $Z_{S1} \text{scalS2_2_S1_1} \otimes X_{S2} * Y_{S1_2_S2_1} - X_{S1_2_S2_1} * Y_{S2}$ $\text{NormeS1scalS2_2_S1_1} \otimes \text{sqrt}(X_{S1} \text{scalS2_2_S1_1}^2 + Y_{S1} \text{scalS2_2_S1_1}^2 + Z_{S1} \text{scalS2_2_S1_1}^2)$ $\text{DistE} \otimes \text{NormeS1scalS2_2_S1_1} / \text{NormeS2}$ Υ' $\text{DistA} \otimes \min(\text{DistB}, \text{DistC}, \text{DistD}, \text{DistE})$ Υ' $\text{Dist2} \otimes \text{sqrt}(((S1X1 - S2X1) * (S1X1 - S2X1)) + ((S1Y1 - S2Y1) * (S1Y1 - S2Y1)) + ((S1Z1 - S2Z1) * (S1Z1 - S2Z1)))$ $\text{Dist3} \otimes \text{sqrt}(((S1X1 - S2X2) * (S1X1 - S2X2)) + ((S1Y1 - S2Y2) * (S1Y1 - S2Y2)) + ((S1Z1 - S2Z2) * (S1Z1 - S2Z2)))$
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$YS1scalS2_1_S1_1 \otimes ZS1 * XS2_1_S1_1 - ZS2_1_S1_1 * XS1$ $ZS1scalS2_1_S1_1 \otimes XS1 * YS2_1_S1_1 - XS2_1_S1_1 * YS1$ $NormeS1scalS2_1_S1_1 \otimes$ $\sqrt{XS1scalS2_1_S1_1 * XS1scalS2_1_S1_1}$ $+ YS1scalS2_1_S1_1 * YS1scalS2_1_S1_1 +$ $ZS1scalS2_1_S1_1 * ZS1scalS2_1_S1_1)$ $DistB \otimes NormeS1scalS2_1_S1_1 / NormeS1$	$Dist4 \otimes \sqrt{((S1X2-S2X1)*(S1X2-S2X1)) + ((S1Y2-S2Y1)*(S1Y2-S2Y1)) + ((S1Z2-S2Z1)*(S1Z2-S2Z1))}$ $Dist5 \otimes \sqrt{((S1X2-S2X2)*(S1X2-S2X2)) + ((S1Y2-S2Y2)*(S1Y2-S2Y2)) + ((S1Z2-S2Z2)*(S1Z2-S2Z2))}$ $Dist \otimes \min(DistA, Dist2, Dist3, Dist4, Dist5)$
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The Normal Distribution

Depending on the uncertainties and taking into account the security areas of each network, the risk of collision can be determined thanks to the Standard normal distribution over a specified threshold distance ($d_{threshold}$) (shown in Figure 14).

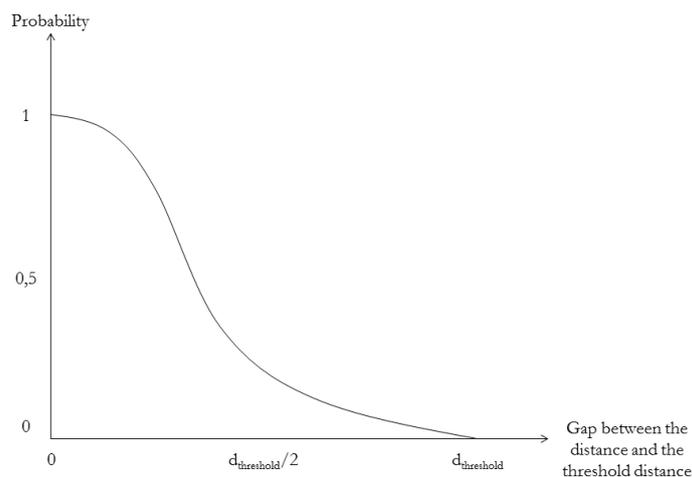


Figure 14: Standard normal distribution representation

The calculation of the threshold distance depends on the elements and their characteristics (like the data precision, the security area,...). Each characteristic could be optional, that's why some of them are noted between square brackets (Table 3).

Table 3: Uncertainty calculations

Category	Threshold distance calculation	Standard normal distribution
Wired – Wired	$d_{threshold} = Radius1 + Radius2 + Precision1 + Precision2 + \text{Max}(SecurityRadius1, SecurityRadius2)$	$\mu = 0$ $\sigma = d_{threshold}/4$
Wired – Element	$d_{threshold} = Radius1 + Precision1 [+ Precision2] + \text{Max}(SecurityRadius1, [SecurityRadius2])$	$\mu = 0$ $\sigma = d_{threshold}/4$
Element – Element	$d_{threshold} = [Precision1] [+ Precision2] + \text{Max}([SecurityRadius1], [SecurityRadius2])$	$\mu = 0$ $\sigma = d_{threshold}/4$