GEORGE BIDDELL AIRY AND HIS CONTRIBUTION TO MAP PROJECTIONS THEORY

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Abstract
Sir George Biddell Airy was an English astronomer, mathematician and physicist with a great interest in geodesy and geophysics. According to Airy's autobiography, he published 518 papers, only two out of which are associated with cartography. In the one published in 1861, Airy was the first to propose the least squared method to azimuthal projection with the least distortion ("minimum-error"). He approached the development of his projection with a goal of having a minimum "total evil" or "total misrepresentation" determined by what he called "Balance of Errors". The present paper explains Airy's method and the need to differentiate between the terms error and distortion in the map projection theory. Airy's second cartographic paper, published in 1858, is rather unknown and deals with his method of approximating a geodetic line using a circular arc on a map produced in the normal aspect Mercator projection.

Keywords*: George Biddell Airy, map projection, minimum distortion, geodetic line*

1. INTRODUCTION

Sir George Biddell Airy was an English astronomer, mathematician and physicist with a great interest in geodesy and geophysics. Many of his achievements are associated with planet orbits, Earth density and solving two-dimensional problems of solid mechanics. In 1830, he calculated dimensions of ellipsoid as Earth's model based on measuring 14 parallel arc lengths and 4 meridian arc lengths (grade measurements). The ellipsoid was named after Airy. In 1851, he established the Prime Meridian (Greenwich), which was internationally accepted in 1884.

Airy was born in Alnwick on July 27, 1801 (Northumberland, England) and he died in Greenwich on January 2, 1892. He was a professor of astronomy and experimental philosophy in Cambridge until 1835, when he became the director of the Greenwich Observatory and served as a Royal Astronomer until 1881. In 1855, he established the contemporary view on isostasy with his hypothesis on the distribution of densities in the upper parts of Earth's crust, in contrast to Pratt's compensation hypothesis.

Airy experimented with aberration of light and proved Fresnel's theory of light. Airy determined Earth's density in 1855. He studied measurement error theory and conducted geomagnetic measurements at the Greenwich observatory. His most significant and most important work is *Figure of the Earth*, published in London in 1849.

Airy's contributions to sciences are best illustrated by a series of terms named after him: Airy disk, Airy point, Airy hypothesis, Airy ellipsoid 1830, Airy's map projection, Airy's theory of isostasy, Airy wave theory, Airy functions Ai(x) and Bi(x) and the differential equation they are based on, Airy-Heiskanen gravity anomaly, Airy-Heiskanen gravity correction, Airy-Heiskanen gravity reduction. Airy also had a crater on Mars named after him. Within it, there is a smaller crater Airy-0, the position of which is defined by the Mars Prime meridian, in the same manner as position of the Airy telescope on Earth is defined by the Greenwich meridian. A crater on the Moon was also named after Airy.

According to his autobiography (Airy 1896), he published 518 papers, only two out of which are associated with cartography (Airy 1858, 1861). The first one is about the approximation of a great circle arc on Mercator's chart by an arc of a circle, and that paper is almost unknown. The second one is a proposal of a new azimuthal projection, which is famous in the map projection theory. The concept was stimulating enough to result in considerable scholarly attention in subsequent textbooks and articles, considering the minimal cartographic use made of the projection (Snyder 1993).

2. AIRY'S METHOD OF REPRESENTING THE PROJECTION OF A GREAT CIRCLE

Airy's paper published in 1858 is rather unknown and deals with his method of approximating a geodetic line using a circular arc on a map produced in the normal aspect Mercator projection of a sphere. The paper is not even mentioned by Snyder in his famous Bibliography of Map Projections (Snyder, Steward 1988). Airy was suggested by a person of the Dutch mercantile marine to develop a method of approximation on Mercators's chart of the projection of a great circle by an arc of a circle, to be swept with an ordinary pair of compasses. In his paper, Airy (1858) explained:

1. Join the first and last point of the course (or the point of departure and the point of arrival) by a straight line; find its middle; erect there a perpendicular to that line, on the side next the equator; and produce it, if necessary, beyond the equator. The centre of the sweep will be on this perpendicular.

2. With middle latitude, enter the Table 1, and take out the corresponding parallel. (It will be remarked that the four first numbers and the last can never come into use.)

Table 1. Middle latitude and the corresponding latitude of the parallel needed for the construction of the approximation of a great circle in the Mercator projection (Airy, 1858)

Middle Latitude.		Corresponding Parallel.	Middle Latitude.		Corresponding Parallel.
5°	Opposite Name	89° 59′ 55″	48°	Opposite Name	21° 42′
10		89 34	50		16 39
15		86 52	52		11 33
20		81 13	54		6 24
22		78 16	56		1 13
24		74 59	58	Same Name	4 0
26		71 26	60		9 15
28		67 38	62		14 32
30		63 37	64		19 50
32		59 25	66		25 9
34		55 5	68		30 30
36		50 36	70		35 52
38		46 0	72		41 14
40		41 18	74		46 37
42		36 31	76		52 1
44		31 38	78		57 25
46		26 42	80		62 51
			85		76 25

3. The centre of the sweep will be the intersection of the perpendicular, (drawn in conformity with the precept in step 1) with the parallel thus found.

4. Fix one point of the compasses in this intersection, and with the other point sweep through the point of departure and the point of arrival; this sweep is the curve required.

Airy calculated the latitude of the parallel needed for the construction of the approximation of a great circle in the Mercator projection by using the following formula

$$\log\tan\left(\frac{x}{2}+45^\circ\right)=\log\tan\left(\frac{\varphi_m}{2}+45^\circ\right)-0.43429448\times \mathrm{nat.cosec}\,\varphi_m \quad (1)$$

where *x* is the needed latitude, φ_m is the given middle latitude, 0.4342448 is the modulus of common logarithms, and nat.cosec means the natural cosecans trigonometric function, which is equal to reciprocal of the sinus function.

Airy lived during a period with no computers, which means tables were important and used frequently. Furthermore, tables with Brigg's logarithms existed, but not for natural logarithms. This is why Airy used Brigg's logarithms. Nowadays, there is no problem of using natural logarithms, and Airy's formula (1) can be simplified into

$$\ln\tan\left(\frac{x}{2}+45°\right)=\ln\tan\left(\frac{\varphi_m}{2}+45°\right)-\frac{1}{\sin\varphi_m}, \quad (2)$$

from where the unknown *x* can be easily computed. Of course, a table like Table 1 is no longer necessary.

Example

Let us suppose we want to travel by boat from Varna to Sevastopol. Geographic coordinates of Varna are 43°13'N, 27°55'E, while coordinates of Sevastopol are 44°36'N, 33°32'E. The distance from Varna to Sevastopol is approximately 476 km. To construct the arc of a circle approximating the great circle on Mercator's chart, we only need the middle latitude φ_m, which is in this case 43°54.5'N. By using formula (2), it is easy to obtain *x* = 31°52'S.

Constructing a corresponding arc of circle would require a large enough compass and a map in a small scale. On the other hand, if we use a software tool for drawing, there is a problem of knowing circle arc coordinates. Airy did not deal with these issues.

3. AZIMUTHAL PROJECTION WITH THE LEAST DISTORTION

In a paper published in 1861, Airy was the first to propose applying the least squares method to azimuthal projection with the least distortion ("minimum-error"). He approached the development of his projection with a goal of having a minimum of "total evil" or of "total misrepresentation" determined by what he called *Balance of Errors*. His Projection by Balance of Errors is neither perspective, conformal, nor equal-area, but it is a compromise appearing very much like the azimuthal equidistant projection, especially if limited to about one hemisphere. Airy's approach was stimulating and resulted in significant attention, appearing in numerous textbooks and papers, even though it was not applied in practice much (Hinks 1912, Maling 1973).

Airy (1861) introduced the term average square length distortion in the entire mapped area, by analogy to the theory of errors. It was not really average square distortion, but sum of squared distortions, which obviously leads to the same results when comparing different projections (Kavrayskiy 1958).

Airy's special approach was to minimize the sum of squared distortions of a linear scale in azimuthal projections in the direction of the radius from the centre of the projection and perpendicular to the direction. In the normal aspect azimuthal projections, they are directions along meridians and parallels.

Airy's principle is founded upon the following assumptions and inferences:

First. The change of area being represented by

$$\frac{\text{projected area}}{\text{original area}}-1 \quad (3)$$

and the distortion being represented by

$$\frac{\text{ration of projected sides}}{\text{ratio of original sides}}-1=\frac{\text{projected length}\times\text{original breadth}}{\text{projected breadth}\times\text{original length}}-1 \quad (4)$$

where the length of the rectangle is in the direction of the vertical (great circle) connecting the rectangle's centre with the centre of reference, and the breadth is transverse to that vertical, these two deformations, when of equal magnitude, may be considered as "equal evils".

Second. As the annoyance produced by a negative value of either of these formulae is as great as that produced by a positive value, we must use some even power of the formulae to represent the real amount of the "evil" of each. Airy took the square.

Third. The "total evil" in the projection of any small part may properly be represented by the sum of these squares.

Fourth. The "total evil" on the entire map may therefore be properly represented by the summation through the whole map of the sum of these squares for every small area.

Fifth. The process for determining the most advantageous projection will therefore consist in determining the laws expressing the radii of almucantars in the projection plane, which will make the "total evil", represented as has been stated, the smallest possible.

Let l and b be the length and breadth of a small rectangle on the Earth's surface, and suppose that the length and breadth of the corresponding rectangle on the map are $l+\Delta l$ and $b+\Delta b$, and neglect powers of Δl and Δb above the first. Then the change of area

$$=\frac{\text{projected area}}{\text{original area}}-1=\frac{(l+\Delta l)(b+\Delta b)}{lb}-1=\frac{\Delta l}{l}+\frac{\Delta b}{b} \quad (5)$$

while the distortion

$$=\frac{\text{projected length} \times \text{original breadth}}{\text{projected breadth} \times \text{original length}}-1=\frac{(l+\Delta l)b}{(b+\Delta b)l}-1=\frac{\Delta l}{l}-\frac{\Delta b}{b}\,. \quad (6)$$

The sum of their squares

$$\left(\frac{\Delta l}{l}+\frac{\Delta b}{b}\right)^2+\left(\frac{\Delta l}{l}-\frac{\Delta b}{b}\right)^2=2\left(\frac{\Delta l}{l}\right)^2+2\left(\frac{\Delta b}{b}\right)^2. \quad (7)$$

Therefore, according to Airy, we may use

$$\left(\frac{\Delta l}{l}\right)^2+\left(\frac{\Delta b}{b}\right)^2 \quad (8)$$

as the measure of the "evil" for each small rectangle.

Let z be the length, expressed in terms of radius, of the vertical (arc of great circle) on Earth connecting the centre of the small rectangle with the centre of reference; ρ the corresponding distance on the map, expressed in terms of the same radius, of the projection of the centre of the small rectangle from the centre of the map; the object of the whole investigation is to express ρ in terms of z. Let the length of a small rectangle on Earth be Δz, the corresponding length on the map $\Delta\rho$. Also let ω be the minute angle of azimuth under which, in both cases, the breadth of the rectangle is seen from the centre of reference or the centre of the map. Then we have

$$l=\Delta z\,,\ l+\Delta l=\Delta\rho\,,\ \Delta l=\Delta\rho-\Delta z \quad (9)$$

$$b=\omega\sin z\,,\ b+\Delta b=\omega\rho\,,\ \Delta b=\omega(\rho-\sin z) \quad (10)$$

$$\left(\frac{\Delta l}{l}\right)^2+\left(\frac{\Delta b}{b}\right)^2=\left(\frac{\Delta\rho}{\Delta z}-1\right)^2+\left(\frac{\rho}{\sin z}-1\right)^2. \quad (11)$$

This quantity expresses the "evil" on each small rectangle. The product of the "evil" by the extent of surface which it affects, omitting the general multiplier ω, is

$$\left[\left(\frac{\Delta\rho}{\Delta z}-1\right)^2+\left(\frac{\rho}{\sin z}-1\right)^2\right]\sin z\Delta z\,. \quad (12)$$

Consequently the summation of the partial "evils" for the whole map is represented by

$$\int\left[\left(\frac{d\rho}{dz}-1\right)^2+\left(\frac{\rho}{\sin z}-1\right)^2\right]\sin z dz\,. \quad (13)$$

This is a case of the Calculus of Variations. Airy (1861) demonstrated that finding the minimum of the integral comes down to solving a second order Euler-Lagrange differential equation and obtained the following general solution:

$$\rho=-2\cot\frac{z}{2}\ln\cos\frac{z}{2}+C_1\cot\frac{z}{2}+C_2\tan\frac{z}{2}\,, \quad (14)$$

where C_1 and C_2 are constant. By setting conditions

$$\rho(0)=0 \text{ and } \frac{d\rho}{dz}(0)=1 \quad (15)$$

Airy reached the final solution

$$\rho(z)=\tan\frac{z}{2}-2\cot\frac{z}{2}\ln\cos\frac{z}{2}\,. \quad (16)$$

Airy continued by comparing his projection with several other azimuthal projections: azimuthal projection equidistant along meridians, equal-area azimuthal, stereographic projection and James projection. Table 2 contains basic data on those projections.

Table 2. Several azimuthal projections and their properties

Equidistant azimuthal	$\rho=z$	$\rho(0)=0$	$\frac{d\rho}{dz}=1$	$\frac{d\rho}{dz}(0)=1$
Equal-area azimuthal	$\rho=2\sin\frac{z}{2}$	$\rho(0)=0$	$\frac{d\rho}{dz}=\cos\frac{z}{2}$	$\frac{d\rho}{dz}(0)=1$
Stereographic	$\rho=2\tan\frac{z}{2}$	$\rho(0)=0$	$\frac{d\rho}{dz}=\frac{1}{\cos^2\frac{z}{2}}$	$\frac{d\rho}{dz}(0)=1$
James's projection	$\rho=\frac{5\sin z}{3+2\cos z}$	$\rho(0)=0$	$\frac{d\rho}{dz}=\frac{5(3\cos z+2)}{(3+2\cos z)^2}$	$\frac{d\rho}{dz}(0)=1$

Considering the third and fifth column in Table 2, Airy's conditions $\rho(0)=0$ and $\frac{d\rho}{dz}(0)=1$ seem completely natural. However, in his review of Airy's 1861 paper, Clarke determined Airy "inadvertently made a mistake" in one of the constants because he did not set integration boundaries and thus did not set correct boundary conditions for solving the Euler-Lagrange differential equation (James, Clarke 1862). Airy made "the mistake" of supposing the constants were completely at his disposal. However, if one seeks the minimum of integral in the interval $[0,\beta]$ then, according to Clarke (James, Clarke 1862) or Young (1920), constants C_1 and C_2 which appear in the solution of the Euler-Lagrange differential equation have to be determined from the condition

$$\frac{d\rho}{dz}(0)=1 \text{ and } \frac{d\rho}{dz}(\beta)=1\,. \quad (17)$$

Taking this into consideration, Clarke reached the following solution

$$\rho(z)=C\tan\frac{z}{2}-2\cot\frac{z}{2}\ln\cos\frac{z}{2}\,, \quad (18)$$

where

$$C = -2\cot^2\frac{\beta}{2}\ln\cos\frac{\beta}{2}. \quad (19)$$

A comparison between the Airy's original projection and the projection corrected by Clarke (named Airy-Clarke's projection) is provided in Table 3.

Table 3. A comparison between the Airy's original projection and the Airy-Clarke's projection

Airy's projection	$\rho(z) = \tan\frac{z}{2} - 2\cot\frac{z}{2}\ln\cos\frac{z}{2}$	$\rho(0) = 0$	$\frac{d\rho}{dz} = \frac{1}{2\cos^2\frac{z}{2}} + \frac{\ln\cos\frac{z}{2}}{\sin^2\frac{z}{2}} + 1$	$\frac{d\rho}{dz}(0) = 1$
Airy-Clarke's projection	$\rho(z) = C\tan\frac{z}{2} - 2\cot\frac{z}{2}\ln\cos\frac{z}{2}$, $C = -2\cot^2\frac{\beta}{2}\ln\cos\frac{\beta}{2}$	$\rho(0) = 0$	$\frac{d\rho}{dz} = \frac{C}{2\cos^2\frac{z}{2}} + \frac{\ln\cos\frac{z}{2}}{\sin^2\frac{z}{2}} + 1$	$\frac{d\rho}{dz}(0) = \frac{1+C}{2}$

For what angle β would Airy-Clarke's projection have the property $\frac{d\rho}{dz}(0) = 1$? It should be $C = 1$, i.e. $2\ln\cos\frac{\beta}{2} + \tan^2\frac{\beta}{2} = 0$. From there, β = 0, which implies that the two projections are always different.

Let us notice that for *z* = 0, equations of Airy's and Airy-Clarke's projection are undetermined. However, it is not difficult to see that if *z* approaches 0, then ρ also approaches 0, i.e. $\lim_{z\to 0}\cot\frac{z}{2}\ln\cos\frac{z}{2} = 0$.

Illustrations of the Airy-Clarke projection can be found in references (James, Clarke 1862, Snyder 1985, 1993).

4. DISCUSSION ON THE FOUNDATIONS OF AIRY'S PROJECTION

Nowadays, map projection theory terminology differs somewhat from the one used by Airy 150 years ago. He distinguished between change of area and distortion. Nowadays, it is common to talk about distortion of areas, distances and angles.

Kavrayskiy (1958) explains that Airy used the following expression as a measure of projection distortion in a given point:

$$\sqrt{(a-1)^2 + (b-1)^2} \; , \quad (20)$$

where *a* and *b* are distortion ellipse semi-axes in a given point. However, Airy first used a different measure of distortion in a given point

$$\left(\frac{a}{b} - 1\right)^2 + (ab-1)^2, \quad (21)$$

and subsequently moved to mathematical analysis and changed the measure with the square of expression (1). Kavrayskiy (1958) demonstrated that values of distortion obtained in these two ways differ by a small quantity of the sum of third and higher order members:

$$\left(\frac{a}{b} - 1\right)^2 + (ab-1)^2 = 2\left[(a-1)^2 + (b-1)^2\right] + III \; , \quad (22)$$

where *III* indicates the sum of third and higher order members.

Laskowski (1998) mentions distortion indicators and distinguishes between area distortion indicator) $ab-1$ and shape distortion indicator $\frac{a}{b}-1$. Since $ab\pi$ is the area of the Tissot indicatrix and $1^2\pi=\pi$ is the area of the original circle, ab is the ratio of these two areas. Therefore, we are able to give ab the geometric interpretation of the area scale factor at a given point on the map and $ab-1$ can be interpreted as the area scale distortion. When $ab=1$, area distortion is zero. The second indicator, $\frac{a}{b}-1$, has a geometric interpretation very similar to the flattening of the distortion ellipse (indicatrix). When $a=b$, the flattening is zero and the indicatrix becomes a circle – indicating zero distortion in shape at that point.

The first member in expression (21) represents approximate squared greatest angle distortion in a given point and the second one represents the square of area distortion in the vicinity of the given point. Numerically, equal values of angle and area distortions were equally undesirable to Airy. According to Kavrayskiy (1958), Airy's assumption is completely arbitrary and can be modified by introducing weights.

Results of comparing estimates of various projections are obviously not going to change if we replace (20) with half of the expression under the root, i.e.

$$\varepsilon^2=\frac{1}{2}\left[(a-1)^2+(b-1)^2\right], \quad (23)$$

thus ε can be described as the average square distortion of lengths in a given point in main directions.

Problems of researching projections with minimum distortion continue to inspire scientists (Snyder 1985, 1993). Several authors defined other measure of average square distortion of projection in a given point or the entire mapped area (Jordan 1875, 1896, Kavrayskiy 1958).

In Clarke's 1862 paper "correcting" Airy's work, he also applied Airy's approach to produce perspective azimuthal projections with the least distortions. Behrmann (1910) proposed an equal-area cylindrical projection with the least distortions, but he did not apply the least squares method. Tsinger (1916) applied the least squares method to conformal and equal-area conical projections. Young (1920) applied the least squares method to various azimuthal and conical projections. Projections with the least distortions were also studied by Miller, Reilly, Stirling, Tobler and others, and almost the entire Snyder's 1985 publication (*Computer-Assisted Map Projection Research*) deals with projections with the least distortions. Researching projections with the least distortions continues to inspire scientists, e.g. Canters (1989, 1991, 2002), Canters and De Genst (1997), Laskowski (1998).

5. INSTEAD OF CONCLUSION

Airy named his projection the Projection by Balance of Errors (Airy 1861). In his paper he used terms such as *evil* or *misrepresentation*. He also uses the term *distortion*, but not in its present meaning. In my opinion, it is not adequate to use the term *error* in the map projection because error usually means something wrong, not nice or even bad. There are many types of error. Errors in science range along a spectrum from those relatively local to the phenomenon (usually easily remedied in the laboratory) to those more conceptually derived (involving theory or cultural factors, sometimes quite long-term). One may classify error types broadly as material, observational, conceptual or discursive (Allchin, 2001).

An *error* (from the Latin *error*, meaning "wandering") is an action which is inaccurate or incorrect. In some usages, an error is synonymous with a mistake (for instance, a cook who misses a step from a recipe might describe it as either an error or a mistake), though in technical contexts the two are often distinguished. In statistics, for instance, "error" refers to the difference between the value which has been computed and the correct value.

As it was explained in chapter 3, according to Clarke (James, Clarke 1862), Young (1920) and many others after them, Airy made an error or a mistake in solving the problem of the Calculus of Variations. This error is something completely different from "errors" in his projection. In order to distinguish a mistake (*errare humanum est*) from inevitable or unavoidable distortions in map projections, I propose not using the term error in the map projection theory. We must be aware that distortions are immanent in any map projection, we must be able to control the amount of distortions in any map, and we need to serve as educators to people who are not aware of the properties of map projections and who believe everything represented on a map, including its mathematical base, is without distortions. If we can agree that a distortion in map projections is misrepresentation, something giving a misleading account or impression, then instead of the Projection by Balance of Errors, it would be better to refer to it as the Projection by Balance of Distortions.

Moreover, it will be shown in a future paper that Airy did not make a mistake in his approach published in 1861. In fact, it was Clarke (James, Clarke 1862), followed by Young and others who made an error!

ACKNOWLEDGMENTS

The author is grateful to Željka Tutek for fruitful discussions on mathematical background of the Calculus of Variation and its applications in the map projections theory.

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